15% (1) In launching a satellite aimed at Mars, it is given that the probability of success is 0.7. If it is decided that satellite launches will be attempted until three(3) successes have been obtained. What is the probability that fewer than 10 attempts will be needed?

10% (2) Given $f(x) = \theta x^{-\theta-1}$, $1 < x < \infty$. Find the probability density function (p.d.f.) $y = \ln x$ and $E(x)$.

20% (3) Assume X and Y are independent random variable, where X has p.d.f $f(x) = 2x$, $0 < x < 1$ and $f(y) = 2(1-y)$, $0 < y < 1$. Find the p.d.f of $X + Y$.

20% (4) A random sample of size n=10 is taken from a population with density function

$$f(x) = \frac{1}{\beta^2} xe^{-\frac{x}{\beta}} \quad \text{for } 0 < x < \infty \text{ and zero elsewhere } (\beta > 0).$$

5% (a) Find the Maximum Liklihood Estimator of β. (call it $\hat{\beta}$).

5% (b) Demonstrate whether $\hat{\beta}$ is unbiased or biased.

15% (c) Find the distribution of $\hat{\beta}$ (Hint: use Moment Generating Function).

20% (5) In tossing a pair of dice independently and seperately. What is the probability of getting a sum of 7 before a sum of 4?

15% (6) Let the joint p.d.f of X and Y be $f(x,y) = \frac{3}{7} x(x+y)$, $0 < x < 1$, $0 < y < 1$, zero elsewhere. Let $U = \min(X, Y)$ and $V = \max(X, Y)$. Find the joint p.d.f of U and V.