一、(20分) Let $X_1, X_2, \cdots, X_n$ denote a random sample from a distribution having the p.d.f. $f(x;p) = p^x(1-p)^{1-x}, x = 0, 1$, zero otherwise. For testing $H_0 : p = \frac{1}{4}$ against $H_1 : p = \frac{1}{2}$,

(1) show that $C = \{(x_1, \cdots, x_n) : \sum_1^n x_i \geq c\}$ is the critical region of a most powerful test. (6分)

(2) If $n = 2$ and $c = 1.5$, compute the probability of type I error and the probability of type II error. (6分)

(3). Use the central limit theorem to find n and c so that approximately $Pr(\sum_1^n X_i \geq c; H_0) = 0.10$ and $Pr(\sum_1^n X_i \geq c; H_1) = 0.80$. (8分)

二、(21分) 某公司品管部門為比較A, B兩牌的零件品質做抽樣檢驗，令隨機變數

$$X_i = \begin{cases} 1 & \text{若A牌第}i\text{個零件為不合格品} \\ 0 & \text{若 〃 〃 〃 為合格品} \end{cases}$$

$$Y_j = \begin{cases} 1 & \text{若B牌第}j\text{個零件為不合格品} \\ 0 & \text{若 〃 〃 〃 為合格品} \end{cases}$$

今分別由兩牌的零件中，獨立的抽出 $n_A = 120$, $n_B = 100$ 的隨機樣本，得20個 $X_i = 1$ 及20個 $Y_j = 1$.

(1) 你認為這二組隨機樣本應服從什麼分配？為什麼？(2分)

(2) 試以上述資料，計算樣本平均數，樣本不偏變異數，樣本變異係數並以此三種數量，比較A, B兩牌零件，何者品質較佳。(4分)

(3) 就B牌的資料，求其母體均值 μ_B 的99%信賴區間 (5分)

(4) 請解釋(3)中，信賴係數99%的涵義 (2分)

(5) 試檢定A, B兩牌的資料是否顯示兩者的機率分配相同 ($\alpha = 0.05$) (5分)

(6) 由(5)是否可判定A, B兩牌零件，何者品質較佳？試與(2)的結論比較，兩者是否有矛盾？為什麼？(3分)

三、(25分) 某地區每月牛肉消費量與月平均氣溫資料如下

ROW (i)	Temp. (X)	Consum. (Y)
1	3	10
2	4	11
3	7	22
4	9	24
5	10	29
6	16	40
7	23	52
8	17	35
9	10	18
10	9	19
11	7	17
12	5	21

氣溫(X) 單位：℃

牛肉消費量(Y) 單位：公噸

$\bar{x} = 10$℃

利用"MINITAB"做簡單線性迴歸分析，得以下 output

(背面仍有題目,請繼續作答)

Predictor	Coef	Stdev	t-ratio	p
Constant	5.042	2.356	2.14	0.058
Temp. (X)	1.979	0.205	9.65	0.000

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	1504.2	1504.2	93.14	0.000
Error	10	161.5	16.1		
Total	11	1665.7			

(1) 牛肉產銷公司相信月平均氣溫每增加一度，則每月牛肉消費量將增加1.6公噸，問以上資料是否足以支持公司想法，試以 $\alpha = 0.05$ 檢定之。 (5分)

(2) 當月平均氣溫為15°C時，試求當月牛肉消費量的95% prediction interval. (6分)

(3) 若二維隨機變數(X, Y)服從 bivariate normal distribution with means μ_1 and μ_2, positive variances σ_1^2 and σ_2^2, and correction coefficient $\rho = 0$.

令
$$W = \frac{\sum_1^n [(X_i - \bar{X})(Y_i - \bar{Y})]}{\sqrt{\sum_1^n (X_i - \bar{X})^2}}$$

試証 conditional distribution of W given $X_1 = x_1, \cdots, X_n = x_n$ is $N(0, \sigma_2^2)$. (4分)

(4) 若令
$$U = \sum_1^n (Y_i - \bar{Y})^2 - W^2$$
由(3)

試証 conditional distribution of W^2/σ_2^2 and U/σ_2^2 given $X_1 = x_1, \cdots, X_n = x_n$ are $\chi^2(1)$ and $\chi^2(n-2)$ respectively. 又，由此所得 $\frac{W\sqrt{n-2}}{\sqrt{U}}$ 的機率分配為何種分配？（註：此機率分配 does not depend upon $x_1, x_2, \cdots, x_n$）. (5分)

(5). 因 $\frac{W\sqrt{n-2}}{\sqrt{U}} = \frac{R\sqrt{n-2}}{\sqrt{1-R^2}}$ （証明省略），利用此特性及上述ANOVA表試檢定 $H_0: \rho = 0$ 是否成立 ($\alpha = 0.05$) (5分)

二、(34分) A traffic engineering use an experimental design to study on traffic delay was conducted at intersections with signals on urban streets. Three types of traffic signals were utilized in the study: (a) pretimed, (b) semi-actuated, and (c) fully actuated. Five intersections were used for each type of signal. The measure of traffic delay used in the study was the average stopped time per vehicle at each of the intersections (seconds/vehicle).

(1). How to collect the data ? Please give a suggestion. (4分)

If the data follow,

ROW	Pretimed	Semi-actuated	Fully actuated
1	37	18	15
2	39	21	10
3	30	19	19
4	37	26	11
5	34	22	15

(2). Point out the response, factor(or factors), levels, treatments and experimental units in this design. (4分)

(3). Write the linear statistical model for this study and explain the model components. (4分)

(4). State the assumptions necessary for an analysis of variance of the data. (3分)

A "MINITAB' output follows,

ANALYSIS OF VARIANCE

SOURCE	DF	SS	MS	F	p
FACTOR	2	(A)	(C)	50.80	0.000
ERROR	12	(B)	(D)		
TOTAL	14	1325.7			

LEVEL	N	MEAN	STDEV
C1	5	35.400	3.507
C2	5	21.200	3.114
C3	5	14.000	3.606

POOLED STDEV = 3.416

(5). Compute the items (A) (B) (C) (D) in the above ANOVA table. (4分)

(6). Compute the 95 % confidence interval of mean of the signal type (a). (5分)

(7). Test the hypothesis of no difference between the mean traffic delay of the signal types (b) and (c) at the 0.05 level of significance. (5分)

(8). Write the normal equations for the data. (5分)

參考資料：$z_{0.2}=0.842$, $z_{0.1}=1.282$, $z_{0.05}=1.645$, $z_{0.025}=1.96$
$z_{0.01}=2.328$, $z_{0.005}=2.576$, $\chi^2_{0.05}(1)=3.84$, $\chi^2_{0.025}(1)=5.02$
$\chi^2_{0.05}(10)=18.31$, $\chi^2_{0.025}(10)=20.48$, $t_{0.05}(4)=2.132$, $t_{0.025}(4)=2.776$
$t_{0.05}(10)=1.812$, $t_{0.025}(10)=2.228$, $t_{0.05}(11)=1.796$, $t_{0.025}(11)=2.201$
$t_{0.05}(12)=1.782$, $t_{0.025}(12)=2.179$.