(10%) 1. Let $U \sim uniform(0,1)$. Find transformation $y = g(u)$ such that $Y = g(U)$ has exponential distribution with pdf $f(y) = \theta e^{-\theta y}$ for $y > 0, \theta > 0$.

(10%) 2. Let X and Y have the joint pdf $f(x,y) = 4e^{-2(x+y)}$, $x > 0, y > 0$. Find the pdf of $X/(X+Y)$.

(30%) 3. Suppose that X and Y have the joint pdf $f(x,y) = cxy$, $0 \le x \le y \le 1$.

(a) Find c so that $f(x,y)$ is a joint pdf.

(b) Find the conditional pdf $f(y|x)$.

(c) Find $E(Y)$.

(10%) 4. Let $X_1, X_2, \ldots$ be independent Bernoulli random variables, $X_i \sim Bernoulli(p)$, and let $\hat{P}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. If $np = \mu > 0$ as $n \to \infty$, find the limiting distribution of $n\hat{P}_n$.

(20%) 5. Consider a random sample of size n from a Poisson distribution, $X_i \sim Poisson(\mu)$.

(a) Find the maximum likelihood estimator (MLE) of $\theta = e^{-\mu}$.

(b) Find the UMVUE of θ.

(20%) 6. Consider independent random samples from two exponential distributions, $X_i \sim exponential(\theta_1)$ and $Y_j \sim exponential(\theta_2)$, $i = 1, \ldots, n$, $j = 1, \ldots, m$.

(a) Derive a $100(1-\alpha)\%$ confidence interval for θ_2/θ_1.

(b) Derive the generalized likelihood ratio (GLR) test of $H_0 : \theta_1 = \theta_2$ versus $H_1 : \theta_1 \ne \theta_2$.