注意事項：一、不得使用具特殊功能之計算機(器)。
二、計算部份應列出計算之過程。
三、參考數值表參見第三、四頁。

一、設 $X_1, X_2, \cdots, X_n$ 表來自一平均數 μ，變異數 σ^2 之常態分配之一大小為 n 的隨機樣本。
(一)若母數 μ 及 σ^2 均未知時，試分別求其最概估計量(Maximum likelihood estimator) $\hat{\mu}$ 及 $\hat{\sigma}^2$。(6%)
(二) $\hat{\sigma}^2$ 是否為母體變異數 σ^2 之一個一致估計量(Consistent estimator)？試驗證之。(5%)
(三)若母數 μ 已知時，母體變異數 σ^2 之最概估計量是否符合"不偏性"(Unbiasedness)？試驗證之。(4%)

二、設兩生產線之產品的重量呈常態分配，今分別自該兩生產線隨機抽取產品，得其重量(公克)資料如右表：

生產線 A	38	40	44	32	36
生產線 B	44	38	42	34	42

依據"MINITAB"統計套裝軟體，得電腦列印如下表所示：

```
TWOSAMPLE T FOR X1 VS X2
     N      MEAN      STDEV    SE MEAN
X1   5     38.00       4.47        2.0
X2   5     40.00       4.00        1.8

95 PCT CI FOR MU X1 - MU X2: (-8.2, 4.2)

TTEST MU X1 = MU X2 (VS NE): T= -0.75  P=0.48  DF=  8

POOLED STDEV =        4.24
```

(一)上表中之"POOLED STDEV = 4.24"、"T = −0.75"及"P = 0.48"係如何求得？試分別列出其計算式。(6%)
(二)若無任何有關兩母體變異數之資訊時，試以 $\alpha = 0.05$ 之顯著水準，檢定兩生產線產品之平均重量是否有差異？(8%)
(三)試就 A 生產線之產品重量資料，推求其母體變異數 σ^2 之 95% 信賴區間。(5%)

三、甲與其女友約定於晚上六時至七時之間在學生活動中心碰面，再一起前往成功廳觀賞電影。若彼此約定只等對方 20 分鐘，逾時不候並取消觀賞電影之約。今設他倆於六時至七時之間到達約定地點的時間為機率獨立，並服從均等分配(Uniform distribution)，試求他倆可能碰面之機率？(8%)

四、欲瞭解大學聯考的數學成績(X_1)與英文成績(X_2)對考入大學後對統計學成績(Y)之影響有多高，隨機抽取15位大一學生之該三項成績如下表所示：

學生序號	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
數學成績 X_1	38	50	40	43	76	70	22	23	28	53	39	50	42	61	35
英文成績 X_2	61	58	59	53	62	91	59	42	49	71	42	70	89	47	48
統計成績 Y	79	83	71	78	85	84	72	63	70	85	62	86	77	86	63

今欲藉上表資料配合複線型迴歸模式，以達成前述目標，並已利用"MINITAB"統計套裝軟體得到下列之電腦列印：

Predictor	Coef	Stdev	t-ratio	p
Constant	50.559	6.546	7.72	0.000
X1	0.3569	0.1095	3.26	0.007
X2	0.1626	0.1148	1.42	0.182

s = 5.745 R-sq = 64.0% R-sq(adj) = 57.9%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	.....	.....	.....	10.65	0.002
Error	.....	.....	.....		
Total	14	1098.93			

SOURCE	DF	SEQ SS
X1	1	636.66
X2	1	66.18

(一) 若所欲估計之模式為 $y=\beta_0+\beta_1 x_1+\beta_2 x_2$，試藉由上列之電腦列印，寫出此複迴歸方程式。(2%)

(二) 在 $\alpha=0.05$ 之顯著水準下，檢定此迴歸模式中是否需要放入自變數 x_2？(4%)

(三) 試求 x_1 不變時之 Y 與 x_2 之偏相關係數 $r_{12\cdot1}$，並據以解釋在(二)中之檢定結果。(4%)

(四) 寫出上列 ANOVA 表中未印出之各數值(即虛線所示部份)。(6%)

(五) 電腦列印之 "R-sq = 64.0%" 稱為什麼？如何由 ANOVA 表中之數值求得？(3%)

五. The number of automobile accidents occurring per day in a particular city is believed to have a Poisson distribution. A sample of 80 days during the past year gives the data shown as follow. Do these data support the belief that the number of accidents per day has a Poisson distribution? Use $\alpha=0.05$. (12%)

Number of Accidents	Observed Frequency (days)
0	34
1	25
2	11
3	7
4	3

六. (a) State the assumptions underlying the analysis of variance for a randomized block design. (2%)

(b) The accompanying data are yields on resistance to stain for three materials (M_1, M_2, and M_3) treated with four chemicals in a randomized block design. (A low value indicates good stain resistance.) Is there evidence of differences in mean resistance among the four chemicals? What would you conclude at the $\alpha=.05$ level of significance? (10%)

(c) Construct a 95% confidence interval for the difference between mean responses for chemicals A and B. (5%)

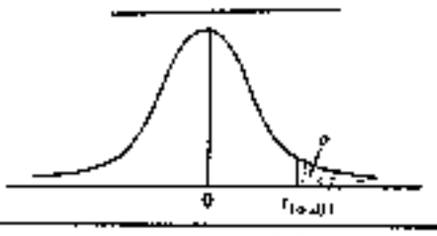
	Material			
Chemical	M_1	M_2	M_3	*Total*
A	5	9	7	21
B	3	8	4	15
C	8	13	9	30
D	4	6	8	18
Total	20	36	28	84

4. Suppose X represents a single observation from the probability density function given by $f(x|\theta) = \theta x^{\theta-1}$, $0 < x < 1$

Find the most powerful test with significance level $\alpha = 0.05$ to test $H_0: \theta = 1$ versus $H_A: \theta = 2$. (10%)

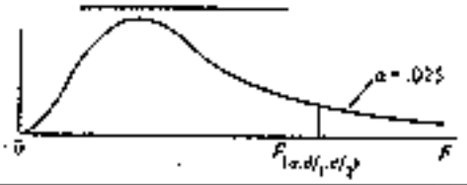
【參考數值表】

Critical values of t

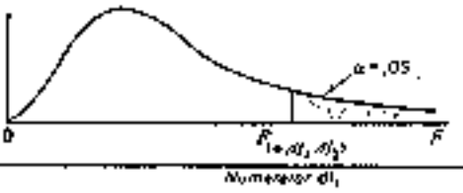


Degrees of Freedom	Upper Tail Areas					
	.25	.10	.05	.025	.01	.005
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
10	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
11	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
12	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
13	0.6938	1.3502	1.7709	2.1604	2.6503	3.0123
14	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
15	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467

Critical values of F



Denominator df_2	Numerator df_1																
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85

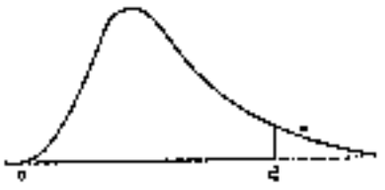


Denominator df_2	Numerator df_1																
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38

Values of e^{-x}

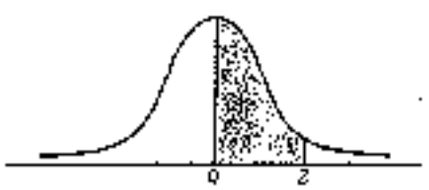
x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.0	1.000	2.5	0.082	5.0	0.0067	7.5	0.00055
0.1	0.905	2.6	0.074	5.1	0.0061	7.6	0.00050
0.2	0.819	2.7	0.067	5.2	0.0055	7.7	0.00045
0.3	0.741	2.8	0.061	5.3	0.0050	7.8	0.00041
0.4	0.670	2.9	0.055	5.4	0.0045	7.9	0.00037
0.5	0.607	3.0	0.050	5.5	0.0041	8.0	0.00034
0.6	0.549	3.1	0.045	5.6	0.0037	8.1	0.00030
0.7	0.497	3.2	0.041	5.7	0.0033	8.2	0.00028
0.8	0.449	3.3	0.037	5.8	0.0030	8.3	0.00025
0.9	0.407	3.4	0.033	5.9	0.0027	8.4	0.00023
1.0	0.368	3.5	0.030	6.0	0.0025	8.5	0.00020
1.1	0.333	3.6	0.027	6.1	0.0022	8.6	0.00018
1.2	0.301	3.7	0.025	6.2	0.0020	8.7	0.00017
1.3	0.273	3.8	0.022	6.3	0.0018	8.8	0.00015
1.4	0.247	3.9	0.020	6.4	0.0017	8.9	0.00014
1.5	0.223	4.0	0.018	6.5	0.0015	9.0	0.00012
1.6	0.202	4.1	0.017	6.6	0.0014	9.1	0.00011
1.7	0.183	4.2	0.015	6.7	0.0012	9.2	0.00010
1.8	0.165	4.3	0.014	6.8	0.0011	9.3	0.00009
1.9	0.150	4.4	0.012	6.9	0.0010	9.4	0.00008
2.0	0.135	4.5	0.011	7.0	0.0009	9.5	0.00008
2.1	0.122	4.6	0.010	7.1	0.0008	9.6	0.00007
2.2	0.111	4.7	0.009	7.2	0.0007	9.7	0.00006
2.3	0.100	4.8	0.008	7.3	0.0007	9.8	0.00006
2.4	0.091	4.9	0.007	7.4	0.0006	9.9	0.00005

Critical Values of χ^2



d.f.	$\chi^2_{.995}$	$\chi^2_{.99}$	$\chi^2_{.975}$	$\chi^2_{.95}$	$\chi^2_{.90}$	$\chi^2_{.05}$	$\chi^2_{.01}$	$\chi^2_{.005}$
1	.0000393	.000157	.000982	.00393	2.706	3.841	6.635	7.879
2	.0100	.0201	.0506	.103	4.605	5.991	9.210	10.597
3	.0717	.115	.216	.352	6.251	7.815	11.345	12.838
4	.207	.297	.484	.711	7.779	9.488	13.277	14.860
5	.412	.554	.831	1.145	9.236	11.070	15.086	16.750
6	.676	.872	1.237	1.635	10.645	12.592	16.812	18.548
7	.989	1.239	1.690	2.167	12.017	14.067	18.475	20.278
8	1.344	1.646	2.180	2.733	13.362	15.507	20.090	21.955
9	1.735	2.088	2.700	3.325	14.684	16.919	21.666	23.589
10	2.156	2.558	3.247	3.940	15.987	18.307	23.209	25.188
11	2.603	3.053	3.816	4.575	17.275	19.675	24.725	26.757
12	3.074	3.571	4.404	5.226	18.549	21.026	26.217	28.300
13	3.565	4.107	5.009	5.892	19.812	22.362	27.688	29.819
14	4.075	4.660	5.629	6.571	21.064	23.685	29.141	31.319
15	4.601	5.229	6.262	7.261	22.307	24.996	30.578	32.801

The standardized normal distribution



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4561	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4705
1.9	.4713	.4719	.4726	.4732	.4736	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4809	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4899	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986