注意事項：計算部份應列出計算之過程
參考數值表參見第三、四頁

1. (1) 為了瞭解水電錶使用者對水、電錶"跑得太快"的糾紛案件，台灣省度量衡檢定所做了隨機抽驗。依據台灣省度量衡檢定所蒐集之資料，一位民眾認為台灣省的水錶之不合格率明顯比電錶之不合格率為高，請問是否有必要進行統計檢定？理由是？ (3 pts)
若有必要，以 0.05 之顯著水準做下列判斷：台灣省的水錶之不合格率是否比電錶之不合格率為高？ (4 pts)

度量衡檢定所對問題水電錶的檢定結果
78年7月1日至79年3月31日

地區	水錶			電錶		
	檢定數	不合格數	不合格率	檢定數	不合格數	不合格率
台灣省	379	62	16.3%	462	25	5.4%

若台南市之消費者也打算估計出該市水錶之不合格率，
(2) 在無任何先驗訊息時，請問應該隨機抽取多少使用者之水錶來檢定才能使台南市水錶之不合格率的 98% 信賴區間之長度為 0.05 ？(3 pts)
(3) 若依據(a)中的訊息，請問應該隨機抽取多少使用者之水錶來檢定才能使台南市水錶之不合格率的 98% 信賴區間之長度為 0.05 ？ (3 pts)
(4) 您的心得是？ (2 pts)

2. A group of consumers collects the following data about the number of defective electrical fuses in each package of 50 fuses from a random sample of 40 packages.

Number of defective fuses:	0	1	2	3	4	5	6
Packages	7	12	10	7	1	1	2

The main interest of this group of consumers is to estimate the probability of **two** defective fuses in a package. They estimated this probability by 10/40 = 0.25. The director of quality-control department thinks this estimate is too high.
(1) Do you think this estimate is too high or too low ? **Why** ? (5 pts)
(2) Could you suggest a better estimate for the director of quality-control department ? Do you need any assumptions ? (15 pts)

3. If the number of minutes a doctor spends with a patient is a random variable having an exponential distribution with the scale parameter $\theta=9$ (which is the mean of this exponential distribution)
(1) Derive the moment generating function for exponential random variable (4 pts)
(2) What are the probabilities that it will take the doctor at least 20 minutes to treat
(i) one patient (2 pts)
(ii) three patients (8 pts)
(write down the steps and the expression of getting this probability)
(3) Do you need any assumptions for computing (2) (i) and (2) (ii) ? (3 pts)
(4) For (2)(ii), could you find the probability by using the table listed in the problemsheet ? (3 pts)

(背面仍有題目,請繼續作答)

4. A marketing researcher studied annual sales of a product that had been introduced 10 years ago. The data are as follows, where X is the coded year and Y is sales in thousands of units:

$(\text{year})_i$	76	77	78	79	80	81	82	83	84	85
X_i	0	1	2	3	4	5	6	7	8	9
Y_i	98	135	162	178	221	232	283	300	374	395

The following SAS output is obtained from the following model $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$.

```
Model: MODEL1              Analysis of Variance
Dependent Variable: Y

                          Sum of        Mean
Source           DF      Squares       Square      F Value      Prob>F

Model             1  87124.37576  87124.37576      387.386      0.0001
Error             8   1799.22424    224.90303
 Total            9  88923.60000

    Root MSE     14.99677     R-square      0.9798
    Dep Mean    237.80000     Adj R-sq      0.9772
    C.V.          6.30646

                        Parameter Estimates

                  Parameter      Standard     T for H0:
Variable  DF      Estimate         Error     Parameter=0    Prob > |T|

INTERCEP   1     91.563636    8.81440719         10.388        0.0001
X          1         ?        1.65108974         19.682        0.0001
```

(1) One student uses the following model $Y_i = \gamma_0 + \gamma_1(\text{year})_i + \varepsilon_i$, and he wonders which ones of the following statements are correct. Please help him to choose the correct ones and give the derivation too. (6 pts)

(a) $\beta_1 = \gamma_1$ (b) $\hat{\beta}_1 = \hat{\gamma}_1$ (c) $\gamma_0 = \beta_0$ (d) $\hat{\beta}_0 = \hat{\gamma}_0$

(e) If $\hat{Y}_i = \hat{\gamma}_0 + \hat{\gamma}_1 (\text{year})_i$ and $\hat{Y}^*_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$ then $\hat{Y}_i = \hat{Y}^*_i$

(f) None of the above

Note that $\hat{\beta}_0$ and $\hat{\beta}_1$ are least square estimators based on Y_i and X_i ;
$\hat{\gamma}_0$ and $\hat{\gamma}_1$ are least square estimators based on Y_i and year_i.

(2) According to the ouput, figure out the estimate of β_1, namely, $\hat{\beta}_1$? (4 pts)

(3) $\hat{\gamma}_0$ = ? Does $\hat{\gamma}_0$ give any relevant information here ? (3 pts)

(4) $\hat{\gamma}_1$ = ? Give the interpretation of $\hat{\gamma}_1$? (3 pts)

(5) What is **determination of coefficient** for this data set ? Interpret **determination of coefficient** for this data set. (3 pts)

(6) Based on this data set, a researcher construct a confidence interval for β_1 with confidence coefficient $1-\alpha_1$ and another confidence interval for β_0 with confidence coefficient $1-\alpha_2$, then the probability that both the confidence intervals will cover the true parameters is (6 pts)

(a) $= (1-\alpha_1)(1-\alpha_2)$ (b) $\geq (1-\alpha_1)(1-\alpha_2)$

(c) $\geq 1-\alpha_1-\alpha_2$ (d) $\leq (1-\alpha_1)(1-\alpha_2)$

(e) $\leq 1-\alpha_1-\alpha_2$

(Please explain **Why** ?)

5. 一位營養學家將比較兩種飲食配方(diet A 與 diet B)對老鼠成長體重增加之效果。每種飲食配方將餵食 10 隻老鼠；此"營養學家"僅有 10 隻老鼠來自同一家族(strain 1) 而另外 10 隻老鼠則來自另一家族 (strain 2)，根據以往經驗知 strain 1 的老鼠比 strain 2 的老鼠為強健且精力旺盛。

(1) 若 strain 1 之老鼠餵食 diet A 之配方，strain 2 之老鼠餵食 diet B 之配方，則此種實驗方式會不會對實驗結果（準確性）有所影響？ (3 pts)

(2) 統計系某同學認為應該將這 20 隻老鼠編號，然後用亂數表將此 20 隻隨機指配餵食飲食配方，你認為此種作法對實驗結果之正確性是否有幫助。(3 pts)

(3) 此位營養學家想知道集區設計(block design)是否可以用來為其進行實驗以蒐集資料 ? 請討論，若可行請協助設計。 (3 pts)

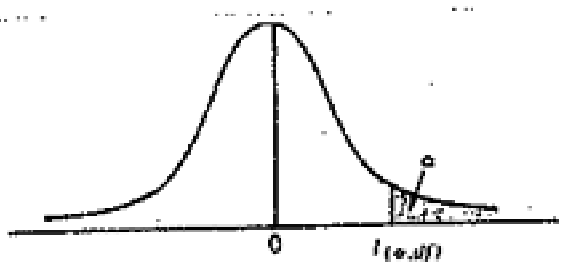
(4) 若此位營養學家欲探討飲食配方與老鼠品種對老鼠成長體重增加之效果，請問應該如何設計一個實驗探討上述主題 ? (3 pts)

(5) 何謂交互作用(interaction) ? 在上述(1), (2), (3), (4) 實驗方法中，哪一個實驗方法能夠用以說明此種現象 ? (3 pts)

(6) 何謂交絡現象(confounding) ? 在上述(1), (2), (3), (4) 實驗方法中，哪一個實驗方法能夠用以說明此種現象 ? (3 pts)

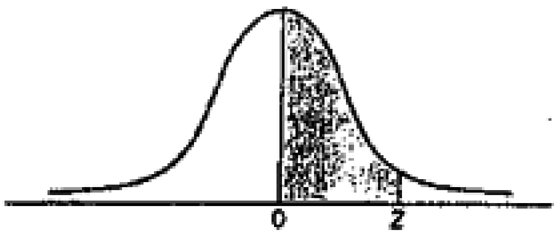
(7) 飲食配方在資料分析中是固定效果(fixed effect) 還是隨機效果(random effect) ? 理由是 ? (2 pts)

【參考數值表】



Degrees of Freedom	Upper Tail Areas .25	.10	.05	.025	.01	.005
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
10	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
11	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
12	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
13	0.6938	1.3502	1.7709	2.1604	2.6503	3.0123
14	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
15	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467

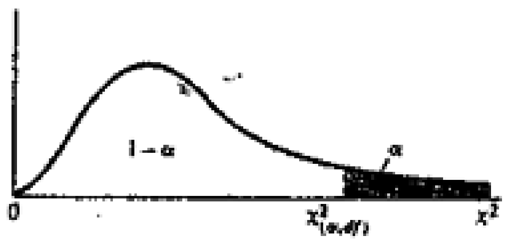
The standardized normal distribution



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986

TABLE E.4 Critical values of χ^2

For a particular number of degrees of freedom, entry represents the critical value of χ^2 corresponding to a specified upper tail area (α)



Degrees of Freedom	Upper Tail Areas (α) .995	.99	.975	.95	.90	.75	.25	.10	.05	.025	.01	.005
1			0.001	0.004	0.016	0.102	1.323	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	0.575	2.773	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	1.213	4.108	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	1.923	5.385	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	2.675	6.626	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	3.455	7.841	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	4.255	9.037	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	5.071	10.219	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	5.899	11.389	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	6.737	12.549	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	7.584	13.701	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	8.438	14.845	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	9.299	15.984	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	10.165	17.117	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	11.037	18.245	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	11.912	19.369	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	12.792	20.489	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	13.675	21.605	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	14.562	22.718	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	15.452	23.828	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	16.344	24.935	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	17.240	26.039	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	18.137	27.141	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	19.037	28.241	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	19.939	29.339	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	20.843	30.435	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	21.749	31.528	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	22.657	32.620	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	23.567	33.711	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	24.478	34.800	40.256	43.773	46.979	50.892	53.672