1. Evaluate the given integral. (20%)

(a) $\int_0^\infty \sqrt{x}e^{-3x}dx$ (b) $\int_0^\pi \frac{\cot x}{\ln \sin x}dx$

(c) $\int_0^{\sqrt{3}/2} sin^{-1}x dx$ (d) $\int_{-\pi}^{\pi} \sin 3x \cos 4x dx$

2. Find the derivative, if exists. (10%)

(a) $y = \tan^{-1}(\sinh x)$ (b) $f(x) = \int_{\ln x}^{e^{e^x}} \ln t \ dt$

3. Determine whether the given series converges absolutely, converges conditionally, or diverges. (10%)

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n}$ (b) $\sum_{n=1}^{\infty} ne^{-n}\sin n^2$

4. Find the sum of the given series, if converges.(10%)

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

5. Evaluate the integral $\int\int_R (x+y)e^{x^2-y^2}dA$ where R is the region bounded by $x^2-y^2=-1$, $x+y=2$, $x^2-y^2=1$ and $x+y=1$. (10%)

6. State(with a brief explanation) whether the following statements are true or false. (10%)
   (a) $\{(1,0,1),(2,1,5)\}$ is a basis for the subspace of vectors in R^3 of the form $(a,b,a+3b)$.
   (b) There are three linearly independent vectors in the R^2
   (c) Any set of three vectors in R^3 that are linearly independent span the space.
   (d) Any two nonzero vectors in R^2 that do not form a basis are collinear.
   (e) The condition that a subset of a vector space contains the zero vector is a necessary and sufficient condition for the subset to be a subspace.

7. Consider the vector $v=(1,3,-1)$ in R^3. Let W be the subspace of R^3 consisting the vectors of the form $(a,b,a-2b)$. Decompose v into the sum of a vector that lies in W and a vector orthogonal to W. (10%)

8. Suppose the product of A and B is the zero matrix: $AB=0$. then the __(1)__ space of A contains the __(2)__ space of B. Also the __(3)__ space of B contains the __(4)__ space of A. Fill in those blank words (1)(2)(3)(4). (10%)

9. (a) If A is similar to B, show that e^A is similar to e^B. First define " similar" and e^A.
   (b) If A has 3 eigenvalues $\lambda = 0, 2, 4$, find the eigenvalues of e^A.
   Then using part (a) explain that *determinant of* $e^A = e^{trace\ of\ A}$. (10%)