Please write down all your work.

1. Find the derivatives dy/dx. 15%

   (a) $y = \ln(|1-e^{2x}|^3)$　(b) $x = \sin xy$　(c) $y = (\tan x)^{tan^{-1}x}$

2. Find the interval of convergence. 10%

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(nx)^n}{n!e^n}$　(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cosh n}{n^2}(x-1)^n$

3. Evaluate the following integrals. 20%

   (a) $\displaystyle\int e^{2x}\sin 3x dx$　(b) $\displaystyle\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} (x^2+y^2)^{3/2} dydx$

   (c) $\displaystyle\int_0^9 \int_{\sqrt{y}}^{3} \sin x^3 dxdy$　(d) $\displaystyle\int_0^{3/2}\int_{\sqrt{2y-y^2}}^{\sqrt{4y-y^2}} xydxdy + \int_{3/2}^{3}\int_{y/\sqrt{3}}^{\sqrt{4y-y^2}} xydxdy$

4. Show that $\displaystyle\int_0^1 \cos\frac{1}{x} dx$ exists. 10%

5. Suppose that g is a continuous real-valued function defined on R, $g(\frac{1}{2}) = 2$, $g(4) = -1$, $\int_{\frac{1}{2}}^{4} g(t)dt = 3$, and $F(x) = \int_1^{x^3} g(\frac{t}{x})dt, \forall x \in R$. Find $F'(1)$. 10%

6. Find the point on the graph of $xy^3z^2 = 16$ that are closest to the origin. 10%

7. A linear transformation $T: R^2 \to R^2$ maps the basis vector $\mathbf{i} = (1,0)$ and $\mathbf{j} = (0,1)$ as follows.

$$T(\mathbf{i}) = \mathbf{i}+\mathbf{j}, \quad T(\mathbf{j}) = 2\mathbf{i}-\mathbf{j}.$$

   (a) Compute $T(3\mathbf{i}-4\mathbf{j})$ and $T^2(3\mathbf{i}-4\mathbf{j})$ in terms of $\mathbf{i}$ and $\mathbf{j}$. 2%

   (b) Determine the matrix of T and of T^2. 3%

   (c) Solve part (b) if the basis is replaced by (e_1, e_2), where $e_1 = \mathbf{i}-\mathbf{j}$ and $e_2 = 3\mathbf{i}+\mathbf{j}$. 5%

8. Let $A = (a_{ij})$ be the 4×4 matrix that has 0 at each diagonal entry and 1 at all other entries; that is, $a_{ij} = 0$ if $i = j$, and $a_{ij} = 1$ if $i \neq j$.

   (a) Find constants a and b such that $A^2 = aA + bI$. 2%

   (b) Prove that A is nonsingular and calculate A^{-1}. 5%

   (c) Determine all the eigenvalues of A (with multiplicity). 3%

   (d) Find 4 independent eigenvectors of A, or else prove that they do not exist. 5%