

編號: E 362 系所: 統計學系

科目: 數學

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

Please write down all your work.

1. Let $f(x) = e^{-1/x^2}$, $x \neq 0$ and $f(0) = 0$. Prove that f is twice differentiable at 0. (10%)2. Find the derivative dy/dx , if exists. (15%)

(a) $y = \ln |1 - \sec^2 x|$

(b) $x = \sin^{-1} xy$

(c) $y = |\sin x|^{\tan x}$

3. Find the following limit, if exists. (15%)

(a) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \sin \frac{(k-1)t}{n}$

(b) $\lim_{n \rightarrow \infty} \sum_{k=1}^n (-1)^k \frac{1}{k}$

(c) $\lim_{n \rightarrow \infty} \sum_{k=1}^n k \sin \frac{1}{k}$

4. Find the following integral $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$. (10%)Hint: Use $x = \pi - y$.5. Show that $\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$ for $|x| \leq 1$. Then find out the value of (10%)

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$$

6. The region R is bounded by $x^2 + xy + y^2 \leq 1$. Prove that (10%)

$$\iint_R e^{-(x^2+xy+y^2)} dx dy = \frac{2\pi}{\sqrt{3}}(e-1).$$

Hint: Let $x = u \cos \alpha - v \sin \alpha$, $y = u \sin \alpha + v \cos \alpha$ for some α , then let $u = a \rho \cos \phi$, $v = b \rho \sin \phi$ for some a and b .

(背面仍有題目, 請繼續作答)

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7. Let $W = \text{span} \{(1, 0, 1), (0, 1, 0)\}$ in R^3 . (10%)

(a) Find a basis for W^\perp .

(b) Show that vectors $(1, 0, 1)$, $(0, 1, 0)$ and the basis for W^\perp from part (a) form a basis for R^3 .

(c) Write the vector $\mathbf{v} = (1, 2, 3)$ as $\mathbf{w} + \mathbf{u}$ with \mathbf{w} in W and \mathbf{u} in W^\perp .

8. Answer each of the following as true (T) or false (F). Justify your answer. (20%)

(a) A diagonal matrix is nonsingular if and only if none of the entries on its main diagonal are zero.

(b) Let $L: R^6 \rightarrow R^{10}$ be a linear transformation defined by $L(\mathbf{x}) = A\mathbf{x}$ for \mathbf{x} in R^6 . If $\dim(\text{range } L) = 3$, then $\dim(\text{Ker } L) = 7$.

(c) The columns of a 5×8 matrix whose rank is 5 form a linearly dependent set.

(d) If A is an $n \times n$ matrix that is row equivalent to I_n , then A is singular.

(e) If A is an 3×3 matrix and $|A| = 3$, then $|\frac{1}{2}A^{-1}| = \frac{8}{3}$.

(f) The linear transformation $L: P_2 \rightarrow P_2$ defined by $L(at^2 + bt + c) = 2at + b$ is one-to-one.

(g) If A is a singular matrix, then A^2 is singular.

(h) $\det(ABC) = \det(BAC)$

(i) If A is an $n \times n$ matrix such that $A^2 = O$, then $A = O$.

(j) A diagonalizable $n \times n$ matrix must always have n distinct eigenvalues.