編號： 366　系所：統計學系　　科目：數學

本試題是否可以使用計算機：□可使用 ，☑不可使用 （請命題老師勾選）

Please write down all your work.

1. Find the following indefinite integrals. (10%)

(a) $\int (2x^2 - x)\tan^{-1} x \, dx$　　(b) $\int \frac{x}{\sqrt{1-2x-x^2}} \, dx$

2. Determine the following integrals converge or diverge. (10%)

(a) $\int_3^\infty \frac{(x-2)\ln x}{x^2} \, dx$　　(b) $\int_0^\infty \frac{(x^2-4x+7)}{(x^2+6)^2} \, dx$

3. Find the following limits. (20%)

(a) $\lim_{x\to 0} \frac{\sin(x^4) - x^4\cos(x^4)}{x^4(e^{2x^4} - 1 - 2x^4)}$　　(b) $\lim_{x\to -\infty} \sqrt{x^2+x} + x$

(c) $\lim_{n\to\infty} \int_0^{2\pi} \frac{\sin nx}{x^2+n^2} dx$　　(d) $\lim_{x\to 0} \frac{\int_0^{x^2} \sin t dt}{x^4}$

4. Find the exact value of the following series. (10%)

(a) $\sum_{n=2}^{\infty} \frac{n(n-1)}{3^n}$　　(b) $1 - \frac{1}{2} + \frac{1}{4\cdot 2!} - \frac{1}{8\cdot 3!} + \frac{1}{16\cdot 4!} - \cdots$

5. Prove that for $0 < x < 1$, (10%)

$$\sqrt{\frac{1-x}{1+x}} < \frac{\ln(1+x)}{\sin^{-1} x} < 1.$$

6. Evaluate (10%)

$$\int_0^1 \int_{-\sqrt{3-3y^2}}^{\sqrt{3-3y^2}} e^{-x^2-3y^2} \, dxdy.$$

（背面仍有題目，請繼續作答）

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7. Suppose q_1, q_2, q_3, q_4 are orthonormal vectors in R^4. Let $A = [q_1\ q_2\ q_3\ q_4]$, $B = [q_1 + q_2\ q_2 + q_3\ q_3 + q_4\ q_4 + q_1]$ and $C = [q_2\ q_3\ q_4\ q_1]$. Find all possible values for the 4 by 4 determinants $\det A$, $\det B$, and $\det A \times \det C$. (10%)

8. Suppose A is a 5 by 3 matrix and Ax is never zero (except when x is the zero vector).
   (a) What can you say about the column of A?
   (b) Show that A^TA is invertible.
   (c) Show that $B = (A^TA)^{-1}A^T$ is a one-sided inversive of A, but not a 2-sided inverse of A. (10%)

9. If A is 3 by 3 symmetric positive definite, then $Aq_i = \lambda_i q_i$ with eigenvalues λ_i and orthonormal einenvectors q_i. Suppose $x = c_1q_1 + c_2q_2 + c_3q_3$. Assume $\lambda_1 \leq \lambda_2 \leq \lambda_3$. What $c's$ will make the ratio x^TAx/x^Tx as large as possible? What is the maximum of the ratio x^TAx/x^Tx? (10%)