

編號: 367 系所: 統計學系

科目: 機率論

本試題是否可以使用計算機:  可使用,  不可使用 (請命題老師勾選)

## 1 Multiple Choice 4% × 8

1. Which of the following statements is(are) true?
  - i. The sum of i.i.d. Exponential random variables is still Exponentially distributed.
  - ii. If any linear combination of several normal random variables is still normally distributed, then they are independent.
  - iii. The sum of i.i.d. Negative Binomial random variables is still a Negative Binomial random variable.A.i B.ii C.iii D.i,ii E.i,iii F.ii,iii G.All H. None
2. Which of the following statements is(are) true?
  - i. If events  $A$  and  $B$  are disjoint, then they are independent.
  - ii. Probability function is a function defined on the sample space  $S$  with range  $[0, 1]$ .
  - iii.  $\{S, \emptyset\}$  is not a  $\sigma$ -field.A.i B.ii C.iii D.i,ii E.i,iii F.ii,iii G.All H. None
3.  $X$  and  $Y$  are independently distributed as the standard normal distribution, which of the following statements is(are) true?
  - i.  $X + Y$  and  $X - Y$  are independent.
  - ii.  $X^2$  and  $Y^2$  are independent.
  - iii.  $X/Y$  follows a  $t$ -distribution.A.i B.ii C.iii D.i,ii E.i,iii F.ii,iii G.All H. None
4. Which of the following statements is(are) true?
  - i. An unbiased estimator which is a function of the MLE, then it is the Best Unbiased Estimator.
  - ii. The size of a hypothesis test is the maximum value of probability of a Type I error.
  - iii. The power function  $K_{\Phi}(\theta)$  is the probability of a type I error when the null hypothesis is true, i.e.  $\theta \in N$ ,  $N$  is the null set.A.i B.ii C.iii D.i,ii E.i,iii F.ii,iii G.All H. None
5. Which of the following statements is(are) true?
  - i. A continuous random variable which is memoryless, then it follows the Exponential distribution.
  - ii.  $F$  distribution does not have a moment generating function.
  - iii. Hypergeometric distribution does not have a moment generating function.A.i B.ii C.iii D.i,ii E.i,iii F.ii,iii G.All H. None

(背面仍有題目,請繼續作答)

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6.  $\bar{X}_n$  and  $S_n^2$  are the sample mean and variance of a random sample with size  $n$  drawn from a population with  $EX^k < \infty, \forall k < \infty$ , which of the following statements is(are) true?
- $\bar{X}_n$  and  $S_n^2$  are independent.
  - $\bar{X}_n$  and  $S_n^2$  are asymptotically independent.
  - $\bar{X}_n$  and  $S_n^2$  are asymptotically uncorrelated.
- A.i B.ii C.iii D.i,ii E.i,iii F.ii,iii G.All H. None
7. Which of the following statements is(are) true?
- If  $X_n \xrightarrow{d} X, Y_n \xrightarrow{d} Y$ , then  $X_n + Y_n \xrightarrow{d} X + Y$
  - Conditional variance is always less or equal to the marginal variance.
  - The maximum value of the density function is one.
- A.i B.ii C.iii D.i,ii E.i,iii F.ii,iii G.All H. None
8. Which of the following statements is(are) true?
- For any population model, there is a sufficient statistic.
  - For any population model, there is a minimum sufficient statistic
  - For any population model, there is a complete sufficient statistic.
- A.i B.ii C.iii D.i,ii E.i,iii F.ii,iii G.All H. None

## 2 Fill in the Blanks 4% × 8

- Let  $\bar{X}$  and  $S^2$  be the sample mean and variance of a random sample of size  $n > 10$  from a normal population with mean  $\mu$  and variance  $\sigma^2$ , what is the value of  $E(\bar{X}S^2)$ ?         A
- Refer to Question 1, what is the value of  $E(\bar{X}/S^2)$ ?         B
- The joint density of  $(X, Y)$  is given as

$$f(x, y) = \frac{x+y}{18}, x, y = 0, 1, 2,$$

$$\text{Cov}(X, Y) = \text{        C        }?$$

- $X$  is a Exponential random variable with mean  $\mu$ , and  $Y = 1 - \exp(-\frac{X}{\mu})$ . What is the distribution of  $Y$ ?         D

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5. The joint moment generating function of  $\mathbf{X} = (X_1, X_2, X_3)$  is

$$M(r, s, t) = (1+r)^{-1}(1+r-s)^{-2}(1+r-s+t)^{-3}$$

where  $r > -1, r-s > -1, r-s+t > -1$ . Now  $U = X_1 - X_2$  and  $V = X_3 - X_2$ , then the joint moment generating function of  $(U, V)$  is  $M(s, t) =$  \_\_\_\_\_ E \_\_\_\_\_?

6. Refer to Question 5,  $\text{Cov}(U, V) =$  \_\_\_\_\_ F \_\_\_\_\_?

7.  $\bar{X}_n$  is a random sample with size  $n$  drawn from a normal population with mean  $\mu$  and variance  $\sigma^2$ , then the asymptotic distribution of  $n(\bar{X}_n - \mu)^2$  is \_\_\_\_\_ G \_\_\_\_\_

8. The joint distribution of a random vector  $\mathbf{X} = (X_1, X_2)'$  is a bivariate normal distribution with mean vector  $\boldsymbol{\mu} = (170, 68)$  and a covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} 400 & 64 \\ 64 & 16 \end{pmatrix}$$

What is the conditional distribution of  $X_1$  given  $X_2 = 72$  \_\_\_\_\_ H \_\_\_\_\_?

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### 3 Problems

1. (12%)  $\mathbf{X} = (X_1, \dots, X_k)'$  is a random vector with conditional distribution given another random vector  $\mathbf{Y} = (Y_1, \dots, Y_k)'$  is

$$\mathbf{X}|\mathbf{Y} \sim N_k(\mathbf{Y}, \Sigma)$$

where  $\Sigma$  is a  $k \times k$  covariance matrix. The distribution of  $\mathbf{Y}$  is  $\mathbf{Y} \sim N_k(\boldsymbol{\mu}, \Lambda)$ , where  $\boldsymbol{\mu}$  is a constant vector with length  $k$  and  $\Lambda$  is a  $k \times k$  covariance matrix. Specify the marginal distribution of  $\mathbf{X}$ .

2. (12%) Bernoulli Process and Poisson Process are two basic ways to describe the process of the occurrences of an event. Several distributions are associated with these two processes, such as Binomial, Negative Binomial, Geometric, Poisson, Exponential, and Gamma distributions. Describe and compare these two processes and the associated distributions.
3. (12%) Suppose  $X$  is a discrete random variable with  $S_X = \{1, 2, 3, 4\}$ . The distribution of  $X$  depends on parameter  $\theta$ , and  $\theta$  takes values  $-1, 0, 1$ . The density function of  $X$  is given as

$x$	1	2	3	4
$f(x; 1)$	0.53	0.3	0	0.17
$f(x; 0)$	0.60	0.20	0.10	0.10
$f(x; -1)$	0.60	0.22	0.18	0

Table 1: Density function of  $X$  with respect to different values of  $\theta$

Find the Likelihood Ratio Test with size 0.15 for testing  $H_0: \theta = 0$  against  $H_a: \theta \neq 0$ .