

Please write down all your work.

1. Find the following limit, if exists. (10%)

$$(a) \lim_{x \rightarrow 0} \cos\left(\frac{1 - |\cos x|}{2x}\right)$$

$$(b) \lim_{n \rightarrow \infty} \sum_{k=0}^n (-1)^k \frac{1}{2k+1}$$

2. Find the following derivatives dy/dx , if exists. (20%)

$$(a) y = (\ln x)^{\ln x}, x = e^2$$

$$(b) \sqrt{x} = \cos^{-1}(xy), x = \frac{\pi^2}{4}$$

$$(c) \tan y = \sinh x, x = \ln 2$$

$$(d) \cosh y = 3^x, x = 2$$

$$\text{Note that } \sinh x = \frac{e^x - e^{-x}}{2} \text{ and } \cosh x = \frac{e^x + e^{-x}}{2}.$$

3. Find the following integrals. (20%)

$$(a) \int_0^{\pi} \sqrt{1 + \cos x} dx$$

$$(b) \int_{\pi/6}^{\pi/4} (1 + \csc x)^2 dx$$

$$(c) \int_0^1 \int_0^{\sqrt{1-x^2}} \sin \sqrt{x^2 + y^2} dy dx$$

$$(d) \int \int_{\Omega} e^{x^2} dx dy, \Omega \text{ is the triangular region bounded by the } x\text{-axis, } 2y = x, x = 2$$

4. Let f be everywhere continuous with $f(1) = a$ and set (10%)

$$F(x) = \int_0^{x^2} \left[t \int_1^t f(u) du \right] dt.$$

Find $F'(x)$, $F'(1)$, $F''(x)$ and $F''(1)$.

5. Set $f(x) = x$, if x is rational; $f(x) = 0$, if x is irrational. Set $g(x) = xf(x)$. (10%)

- Is f continuous at 1?
- Is f continuous at 0?
- Is f differentiable at 0?
- Is g continuous at 0?
- Is g differentiable at 0?

(背面仍有題目,請繼續作答)

本試題是否可以使用計算機： 可使用， 不可使用（請命題老師勾選）

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6. Let $a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $c = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $d = \begin{bmatrix} -7 \\ 2 \\ 2 \end{bmatrix}$. (10%)

- (a) (2%) Give d as a linear combination of a , b and c .
- (b) (6%) By using Gram-Schmidt, orthogonalize a , b and c to get orthonormal vectors q_1 , q_2 and q_3 .
- (c) (2%) Give the change of basis matrix from the basis a , b and c to the basis q_1 , q_2 and q_3 .

7. Let A be a 3×4 matrix such that (10%)

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 - x_3 + x_4 \\ x_1 + 2x_2 - x_4 \\ x_1 + x_2 - 3x_3 + 3x_4 \end{bmatrix}$$

Find the column space and null space s of A with bases and dimensions.

8. If you know that $\det A = \begin{vmatrix} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{vmatrix} = 6$, find the determinants of B , C and D . (10%)

$$\det B = \begin{vmatrix} \text{row1} + \text{row2} + \text{row3} \\ \text{row1} + \text{row2} \\ \text{row1} \end{vmatrix} \quad \det C = \begin{vmatrix} \text{row1} - \text{row3} \\ \text{row2} - \text{row1} \\ \text{row3} - \text{row2} \end{vmatrix}$$

$$\det D = \begin{vmatrix} 1 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 1 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 1 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 1 \end{vmatrix}$$