I. (40 points) Suppose that random variable X takes values -1, 0, 1 with respective probabilities $p_1 = \theta^2$, $p_2 = 2\theta(1-\theta)$, $p_3 = (1-\theta)^2$ given by the Hardy-Weinberg proportions, $0<\theta<1$. A sample size of n, $X_1, X_2, .., X_n$, are obtained.

(a). Find the likelihood function.

(b). Find the maximum likelihood estimator (MLE) T of θ.

(c). Let H_0: $\theta=\theta_0$ vs H_1: $\theta>\theta_0$. Based on minimal sufficient statistic, obtain the uniformly most powerful test (UMPT) at level α.

(d). Find E(T) and Var(T).

II. (24 points) A random variable X with double exponential distribution has the following *p.d.f.*

$$f_{\mu,\lambda}(x) = \frac{1}{2}\lambda e^{-\lambda|x-\mu|}, x \in R, \mu \in R, \lambda > 0.$$

Suppose a random sample $X_1, X_2, .., X_n$ is available, $n=2m+1$, m is positive integer.

(a). For λ fixed, plot the likelihood function for μ. For simplicity, you can take $m=1$.

(b)(continued). Find the maximum likelihood estimator for μ.

(c). If now $\mu = \mu_0$ is known, but λ is unknown, construct a 100(1-α)% confidence interval for λ based on the minimal sufficient statistic, $0< \alpha <1$.

III. (36 points) In a two-color microarray experiment, the gene expression level is usually represented as R/G, where R is the fluorescence intensity of Cye 5 and G is the fluorescence intensity of Cye 3. It is a common practice that both R and G follow lognormal distributions, and both are correlated, although its association is mild if the experimental process is under control. Suppose (*ln*R, *ln*G) follows a bivariate normal with mean μ_1, μ_0, variance σ_1^2, σ_0^2 and correlation ρ. Let M=*ln*R - *ln*G, the differential gene expression level, and A = (*ln*R + *ln*G)/2, the abundance of mRNA.

(a). Find E(R)=μ_R and Var(R)=σ_R^2, and show that E(R) has something to do with the Var(R) is a function of E(R)

(b). If $\sigma_R^2 = f(\mu_R)$, where $f(.)$ is a function with continuous derivative, find a transformation of R, g(R), so that Var(g(R)) is roughly free of μ_R.

(c). Find the joint distribution of (M, A).

(d). Find E(M|A), the conditional expectation of M given A. Under which condition(s) that A has nothing to do with M?