Questions in 3 Parts: Total 100 points ($= 20 + 40 + 40$)

I. True or False (20 points = 10 questions × 2 points/question)
Write O for True and X for False.

1. The median of an uniform distribution equals to one-half of its range.
2. Histogram is appropriate for displaying the test scores of a 7 students' class.
3. The sum of residuals from the fitted model $\widehat{y} = \widehat{\beta}_1 x_1 + \widehat{\beta}_2 x_2$ is always equal to zero.
4. The *sign* of β in linear model $y = \alpha + \beta x + \epsilon$ can be obtained from the *sign* of the correlation coefficient of y and x.
5. We have $\Pr(X \leq x) = \Pr(X \geq n - x), x = 0, 1, \ldots, n$, for $X \sim Binomial(n, p)$.
6. A hypothesis being rejected at 5% significance level implies that it will also be rejected at 10% significance level.
7. For $X \sim Normal(\mu, \sigma^2)$, we have $\Pr(X \leq -3) = \Pr(X \geq 3)$ since the shape of normal distribution is symmetric.
8. It is not appropriate to use stem-and-leaf plot to display Calculus final scores of 3,000 freshmen.
9. The standard deviation is always smaller than the mean of a normal distribution.
10. For $X \sim Normal(3, 2^2)$, we have density function $f_{Normal}(\tilde{x}) = 1/\sqrt{8\pi}$ where $\tilde{x}$ is the median.

II. Fill-in Blanks (40 points = 10 blanks × 4 points/blank)
Specify the number of each blank.
Numerical answer should be rounded to the 2nd decimal place.

- For $X \sim Normal(\mu, \sigma^2)$,
  - in order to have $\Pr(X \leq 0) \leq 0.001$, we need to have $\mu/\sigma \geq \boxed{1}$.
  - suppose $\sigma^2 = 100$, in order to have

$$\Pr\left(\mu - 5 \leq \overline{X} \leq \mu + 5\right) \geq 0.99$$

    we need to have sample size $n \geq \boxed{2}$.
  - with random sample $X_1, X_2, \ldots, X_{21}$, having

$$\sum_{i=1}^{21} X_i = 616 \quad \text{and} \quad \sum_{i=1}^{21} X_i^2 = 19336,$$

    an equal-tailed 90% confidence interval for σ^2 is $\left(\boxed{3}, \boxed{4}\right)$.

(背面仍有題目,請繼續作答)

- with available data of size $n = 16$, having

$$\overline{X} = \frac{1}{16}\sum_{i=1}^{16} X_i = 28.94 \quad \text{and} \quad S^2 = \frac{1}{15}\sum_{i=1}^{16}(X_i - 28.94)^2 = 77.53,$$

the 10th percentile: $x_{.10}$ can be estimated by $\widehat{x}_{.10} = \boxed{5}$ and the probability up to 40.22: $F_{Normal}(40.22)$ can be estimated by $\widehat{F}_{Normal}(40.22) = \boxed{6}$.

- For random sample $X_1, X_2, \ldots, X_{25}$ from $Normal(\mu_X, 5^2)$, and, independently, $Y_1, Y_2, \ldots, Y_9$ from $Normal(\mu_Y, 2^2)$,
  - $7\overline{X} - 3\overline{Y} \sim Normal\left(\boxed{7}, \boxed{8}\right)$;
  - for testing null hypothesis: $\mu_X - \mu_Y = 5$ versus alternative: $\mu_X - \mu_Y \neq 5$, the test statistic is $\boxed{9}$.

- For $X \sim Exponential(\theta = \text{mean})$, having density function $f_{Exponential}(x;\theta) = \exp(-x/\theta)/\theta$, the parameter θ is the $\boxed{10}$th percentile.

III. Derivation and Calculation (40 points = 4 questions × 10 points/question)

Show your work!

- For random sample $X_1, X_2, \ldots, X_n$ from a distribution with discrete density function:

$$f_X(x) = \begin{cases} \theta, & \text{for } x = 1, \\ 1-\theta, & \text{for } x = 3, \\ 0, & \text{otherwise.} \end{cases}$$

for the parameter θ,

(a) derive the (ordinary) least squares estimator $\widehat{\theta}_{LS}$.

(b) derive the maximum likelihood estimator $\widehat{\theta}_{ML}$.

- Let μ_X and σ_X^2 denote the mean and variance, respectively, of random variable X;

(c) for $X \sim Exponential(\theta = \text{mean})$, derive the expression of

$$\Pr\left(|X - \mu_X| \leq k\,\sigma_X\right), \quad \text{for } k \geq 1.$$

(d) for $X \sim Uniform(a = \min, b = \max)$, find the value of k_c such that

$$\Pr\left(|X - \mu_X| \leq k\,\sigma_X\right) = 1, \quad \text{for } k \geq k_c.$$

Tables

Quantile of $Normal(0,1)$ Distribution: $z_p = F^{-1}_{Normal}(p;0,1)$

p	.9	.95	.975	.99	.995	.999
z_p	1.282	1.645	1.960	2.326	2.576	3.090

Quantile of χ^2_{df} Distribution: $\chi^2_{df,p} = F^{-1}_{\chi^2}(p;df)$

	p							
df	.01	.025	.05	.1	.9	.95	.975	.99
19	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19
20	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57
21	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93
22	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29