系所組別： 統計學系
考試科目： 數理統計   考試日期：0308，節次：2

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Multiple Choice 4% × 10=40%

For each question, there is one and only one appropriate answer.

1. Which of the following statements is/are true?

   i. Suppose that X, Y and Z are independent random variables, then $X + Y$ and Z has to be independent.

   ii. Let $\bar{X}_n$ and S_n^2 be the sample mean and variance of an i.i.d. sample $X_1, X_2, \ldots, X_n$, then $\bar{X}_n$ and S_n^2 are asymptotically uncorrelated but not asymptotically independent.

   A. i B. ii C. i and ii D. Neither one.

2. X and Y are independently and identically distributed as standard normal distribution and $Z = X/Y$, which of the following statements is/are true?

   i. The expected value of Z is 1.

   ii. Z does not have a moment generating function.

   A. i B. ii C. i and ii D. Neither one.

3. Which of the following statements is/are true?

   i. If the minimal sufficient statistic is not complete, then we can conclude that there is no complete sufficient statistic.

   ii. X, Y, and Z are three random variables. If X is independent from Y and also Y is independent from Z, then X is independent from Z.

   A. i B. ii C. i and ii D. Neither one.

4. Which of the following statements is/are true?

   i. Minimal sufficient statistic definitely exists.

   ii. If a model has a complete sufficient statistic, suppose that there is also an unbiased estimator of $\tau(\theta)$, then there is a best unbiased estimator (also known as UMVUE) of $\tau(\theta)$ and it has to be a function of the minimal sufficient statistic.

   A. i B. ii C. i and ii D. Neither one.

(背面仍有題目,請繼續作答)

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5. Which of the following statements is/are true?

   i. Suppose that $T(\mathbf{X})$ is the Maximum Likelihood Estimator (MLE) of $\tau(\theta)$, then T^2 is the MLE of τ^2.

   ii. If $A \subset B$ then $P(A) < P(B)$.

   A. i　B. ii　C. i and ii　D. Neither one.

6. Which of the following statements is/are true?

   i. Consider there are N balls in an urn, in which the proportion of red balls is p, and one would like to select n out of these N ball with simple random sampling without replacement. Let X be the number of red balls selected, then the range of X is $\{0, 1, \ldots, n\}$.

   ii. If a random vector $\mathbf{X} = (X_1, \ldots, X_k)$ follows a k-dimensional multinomial distribution, then the marginal distribution of each X_i is a binomial distribution.

   A. i　B. ii　C. i and ii　D. Neither one.

7. Which of the following statements is/are true?

   i. If $P(A \cap B \cap C) = P(A)P(B)P(C)$, then A, B, and C are independent events.

   ii. If X is independent from Y, then $h(X)$ is independent from $g(Y)$ for any function h, g.

   A. i　B. ii　C. i and ii　D. Neither one.

8. Which of the following statements is/are true?

   i. Let $X_1, X_2, \ldots$ be a sequence of random variable with $\mathrm{E}(X_n) = \mu_n$ and $\mathrm{Var}(X_n) = \sigma_n^2$, if $X_n \xrightarrow{P} a$, then we have $\mu_n \longrightarrow a$ and $\sigma_n^2 \longrightarrow 0$.

   ii. For any subset A of the universal set $\mathcal{S}$, $\{A, A^c, \emptyset, \mathcal{S}\}$ has to be a σ-field.

   A. i　B. ii　C. i and ii　D. Neither one.

9. Which of the following statements is/are true?

   i. If $X_n \xrightarrow{d} Z \sim N(0, 1)$ and $Y_n = X_n^2$, then $Y_n \xrightarrow{d} W \sim \chi_1^2$.

   ii. The power function $K_\Phi(\theta)$ is the probability that the null hypothesis would be rejected when the null hypothesis is not true.

   A. i　B. ii　C. i and ii　D. Neither one.

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10. Which of the following statements is/are true?

　i. If the MLE is also unbiased, then it is the best unbiased estimator.

　ii. One of the famous properties of the Exponential distribution is that it is memoryless, but a memoryless continuous random variable which is not necessarily to be an Exponential one.

　A. i　B. ii　C. i and ii　D. Neither one.

Fill in the Blanks 5% × 6=30%

Fill in blanks with your best answer.

1. Let (X, Y) be uniformly distributed on the triangle bounded by $y = 0$, $x+y = 1$, and $x - y = -1$. If $Z = X^2 + Y^2$, then $P(Z < 0.5) = $ _____A_____. In addition, $P(X = Y) = $ _____B_____.

2. Let $\bar{X}_n$ be the sample mean computed from a sample of size n from a distribution with mean μ and variance σ^2, then the asymptotic distribution of $n^{1/2}(\bar{X}_n^2 - \mu^2) \xrightarrow{d}$ _____C_____.

3. Let X_i, $i = 1, \ldots, n$ be independently distributed as Poisson distribution with mean $i\theta$, then the information bound for the variance of the unbiased estimators of θ^2 is _____D_____. In addition, the UMVUE of θ is _____E_____.

4. X has Exponential distribution with mean 1, then the distribution of $V = e^{-X}$ is _____F_____.

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Problems 30%

For full credit, you have to provide the detailed process.

1. (10%) Let (X, Y) have a trinomial distribution with parameters $n = 2$ and $\mathbf{p} = (p_1, p_2)$, find the *most powerful test* of size-0.16 of $H_o : p_1 = p_2 = 0.4$ against $H_a : p_1 = 0.2, p_2 = 0.6$.

2. (10%) If the conditional distribution of X given Y is

$$X|Y \sim N(Y, \sigma^2)$$

and the distribution of Y is

$$Y \sim N(\mu, \eta^2).$$

Please find the marginal distribution of X.

3. (10%) John is the CEO of Dummy Automobile Co. Ltd., which is the largest in the industry. Dummy is so large that people describe Dummy's numbers of production lines to words like "infinite". To minimize impact from the recent economic crisis, John points out the key lies in the betterment of quality control processes, which would in turn lower number of flawed products. Specifically, he decides whenever there is a breakdown in any production line, all other production lines will be stopped, until the cause of the problem is nailed. This is to ensure the same problem will not happen again. Suppose that the occurrences of breakdown of each production line are independent and follows a Poisson process with common rate λ, do you think John's strategy will work? Support your answer with rigor reason(s).

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