※ 考生請注意:本試題 □可 ☑不可 使用計算機

1. Find the following derivatives. (10%)

  (a) $\frac{d}{dx}\sin(e^{-x})$ at $x = 0$ (b) $D_x \ln|\cos 2x|$ at $x = \frac{\pi}{2}$

2. Find the following integrals. (20%)

  (a) $\int_{1/e}^{e^2} \cos(\pi \ln x)\, dx$ (b) $\int_0^{\pi/2} \sqrt{1 + \cos x}\, dx$

  (c) $\int_0^1 \int_x^1 e^{y^2}\, dy\, dx$ (d) $\int\int_{\{y^2 + z^2 \le 16\}} (4 - \sqrt{y^2 + z^2})\, dA$

3. Find the following limits. (10%)

  (a) $\lim_{n \to \infty} \sum_{k=1}^n \frac{(k-1)^3}{n^4}$ (b) $\lim_{t \to 0} \frac{1}{t} \int_x^{x+t} \sqrt{1 + \cos^2 u}\, du$

4. Determine whether the following series converge or not. (10%)

  (a) $\sum_{k=2}^\infty \frac{2^k k!}{(2k+1)!}$ (b) $\sum_{k=2}^\infty \frac{1}{(\ln k)^{\ln k}}$

5. Let $F(x) = \int_1^{|x|} \frac{e^t}{t+1}\, dt$. What is $F'(-4)$? (10%)

6. Find the volume of the ellipsoid $x^2/4 + y^2/4 + z^2/3 = 1$. (10%)

7. Let $A = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -4 \\ -3 \\ 3 \\ 3 \end{bmatrix}$ (20%)

  (a) Find the rank of A and also of $A^T A$ and also of AA^T. (5%)

  (b) Find a basis of the nullspace of $A^T A$. (5%)

  (c) Find an orthonormal basis for the column space of A. (5%)

  (d) Then compute the projection of $\mathbf{b}$ onto the column space. (5%)

（背面仍有題目,請繼續作答）

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8. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ 　　　　　　　　　　(10%)

（a）Is A singular? Why? (3%)

（b）What is the sum of the eigenvalues of A? (3%)

（c）Find the eigenvector of A corresponding to the eigenvalue 3. (4%)