

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、選擇題 50 分(每題五分)

1. If  $X$  is  $N(\mu, \sigma^2)$ , then  $E(|X - \mu|) = ?$  (a)  $\sigma$  (b)  $\sigma\sqrt{2/\pi}$  (c)  $\sigma\sqrt{2\pi}$  (d) 0
2. Toss three coins. Let  $X$  denote the number of heads on the first two and  $Y$  denote the number of heads on the last two. Find the conditional expectation  $E(Y|X=1) =$  (a) 0.5 (b) 1 (c) 1.5 (d) 2
3. Suppose you calculate a correlation between age of head of household and household income. You find a correlation coefficient of 0.55. Approximately what percent of income is "explained" by age? (a) About 74% (b) About 55% (c) About 30% (d) Unknowable from information given.
4. Which of the following statements regarding to hypothesis testing are correct?
  - (a) p-value increases as  $\alpha$  increases
  - (b) Power of a test is the probability of rejecting the null hypothesis when it is false
  - (c)  $\beta$  increases as  $\alpha$  increases
  - (d) p-value of a test is the probability of rejecting the null hypothesis when it is true.
5. Suppose that a point  $(X, Y)$  is chosen at random from the circle  $S$  defined as follows:
 
$$S = \{(x, y) : (x-1)^2 + (y+2)^2 \leq 9\}$$
 then  $\Pr(Y > 0 | X = 2) =$ 
  - (a)  $\frac{2-\sqrt{2}}{4}$  (b)  $\frac{4-\sqrt{2}}{6}$  (c)  $\frac{4-\sqrt{3}}{6}$  (d)  $\frac{1}{4}$
6. Suppose that  $X$  has a  $t$  distribution with  $n$  degrees of freedom ( $n > 2$ ).  $\text{Var}(X) =$ 
  - (a)  $\frac{2}{3n}$  (b)  $\frac{n+1}{4n-3}$  (c)  $\frac{n}{n-2}$  (d)  $\frac{2}{n+2}$
7. If  $\lambda$  is 5 occurrences per ten minute time interval, then if we wished to analyze the number of occurrences per hour, we would use an adjusted  $\lambda$  of \_\_\_\_
  - (a) 5 (b) 60 (c) 30 (d) 10

8. Four percent of the customers of a mortgage company default on their payments. A sample of five customers is selected. What is the probability that exactly two customers in the sample will default on their payments?
- (a) 0.2592 (b) 0.0142 (c) 0.9588 (d) 0.7408
9. A dormitory on campus houses 200 students. 120 are male, 50 are upper division students, and 40 are upper division male students. A student is selected at random. The probability of selecting a lower division student, given the student is female, is :
- (a)  $7/8$  (b)  $7/15$  (c)  $2/5$  (d)  $7/20$
10. A null hypothesis was accepted at the 0.10 level of significance. If the level of significance were changed to 0.05 and the same sample results were obtained, what decision should be made?
- (a) reject the null hypothesis  
 (b) do not reject the null hypothesis  
 (c) cannot be determined without further information  
 (d) all of the above

## 二、非選擇題 50 分

1. (10%) The probability density function of  $\lambda$  is given by

$$f(x) = \begin{cases} a(1-x^2), & \text{if } -1 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

the variance  $Var(X) = \underline{\hspace{2cm}}$ .

2. (10%) Let  $X_1, X_2, \dots$  be i.i.d. (identically independent distributions)  $U(0, \theta)$  random variables. (i.e.

p.d.f of  $X_1$  is  $f(x|\theta) = \frac{1}{\theta} 1_{(0 < x < \theta)}$ ). Let  $Y_1 = \min(X_1, \dots, X_n)$ , find the limiting distribution of

$Z_n = nY_1$  as  $n \rightarrow \infty$ .

3. (20%) Consider a random variable  $X$  with density function

$$f(x) = \begin{cases} \frac{1}{5}, & 0 \leq x \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

(a) (10%) Find  $\mu = E(X)$  and  $\sigma^2 = E(X - \mu)^2$ .

(b) (10%) Demonstrate that Chebyshev's theorem holds for  $k = 1.5$ .

(Hint:  $P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - 1/k^2$ .)

4. (10%) Find the value of

(a) (5%)  $\int_{-\infty}^{+\infty} e^{-\frac{1}{4}(x-3)^2} dx =$

(b) (5%)  $\sum_{x=0}^{\infty} \frac{(4x^2 - 3x)2^x}{x!} =$