1. Show by example that $\lim_{x\to c}[f(x)+g(x)]$ can exist even if $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ do not exist. (10%)

2. Find the critical points and classify all the extreme values of
$$f(x) = x^2 - 2|x| + 2, \quad x \in [-\tfrac{1}{2}, \tfrac{3}{2}]$$ (10%)

3. Evaluate $\iint_{\Omega} (3xy^2 - y)\,dx\,dy$, Ω the region between $y=|x|$ and $y=-|x|$, $x \in [-1, 1]$. (15%)

4. A motorcyclist riding along the road $y=x^2$ is at point (a, b) when she sees a deer standing at point $(1, -4)$ illuminated by her headlight. Find (a, b). (10%)

5. Let R denote the region bounded below by the x-axis and above by the graph of $y = \frac{1}{x^2+1}$. Find the volume generated by rotating R around the x-axis. (10%)

6. Evaluate (a) $\lim_{x\to\infty} e^{-x^2}\int_0^x e^{t^2}dt$ (b) $\lim_{h\to 0}\frac{\sqrt{x+h+5}-\sqrt{x+5}}{h}$ (10%)

7. Evaluate (a) $\int \frac{3x+5}{x^4+x^2}\,dx$ (b) $\int \tan^3 x \sec^3 x\,dx$ (10%)

8. A manufacturer has \$60,000 to spend monthly for labor, interest, and raw materials. If x, y, and z are the amount (in dollars) spent monthly on these resources, respectively, then the monthly profit (in thousands of dollars) is given by $P(x, y, z) = xz + yz + xy$.
How much should be allocated monthly for each resource to maximize the profit? (15%)

9. Expand $f(x) = \frac{4}{4+x^2}$ into a power series in powers of x. Find the interval of convergence of the series. (10%)