

注意:未寫明演算過程者不予計分

1. Find the following values, if exist. (40%)

(a) $\int_0^2 \sqrt{4-x^2} dx$

(b) $\int_0^\infty \frac{1-\cos x}{x^2} dx$

(c) $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x + \cos x}{\sin x + \tan x} dx$

(d) $\int_{-2}^2 \frac{1}{|1-x^2|} dx$

(e) $\int_0^\infty e^{-x^2} dx$

(f) $\lim_{x \rightarrow \infty} (x^2 + 1)^{\frac{1}{x}}$

(g) $\int_{\sqrt{e}}^{e^2} \frac{\sin^2(\ln t)}{t} dt$

(h) $D_x \int_{\sqrt{x}}^{x^2} \frac{\sin u}{u} du$

2. Determine whether the series converges or diverges. (10%)

(a) $\sum_{k=2}^{\infty} \frac{(2k+1)!}{k^2(k+1)!}$

(b) $\sum_{k=2}^{\infty} \frac{1}{\sqrt{k}(\ln k)^3}$

3. Let $f(x) = |x^2 - 1|$. Calculate $f'(x)$ and $f''(x)$. Specify the domains of all three functions. (10%)

4. Find the area bounded by the graphs of the equations $y = x - 6$ and $y^2 + 4 = x$. (10%)

5. The marginal revenue of a small firm is given by $M_R(x) = 125 + 200x(x^2 + 1)^{-1}$. How much will revenue increase if production is increased from 99 items per day to 143 items per day. (10%)

6. The region bounded by the graphs of $y = 0$, $x = e$ and $y = \ln x$ is revolved around the y -axis. Compute the volume of the resulting solid. (10%)

7. Suppose a company earns a daily profit of $P(x, y) = -3x^2 - 2y^2 + 4xy + 50x + 64y - 600$ dollars if it uses x workers and a capital outlay of $\$y$ thousand. How many workers and how much capital should the company use to maximize the daily profit? (10%)