

注意：答案務必要寫明演算過程且化簡，否則不予計分

1. Truth or false. (6%)
 - (a) In general, $\lim_{x \rightarrow a} f(x) = f(a)$.
 - (b) If $\lim_{x \rightarrow a^+} f(x) = L_1$, $\lim_{x \rightarrow a^-} f(x) = L_2$, and $L_1 = L_2$, then f is continuous at $x = a$.
 - (c) If f is continuous at $x = a$, then it is differentiable at $x = a$.
2. Find the following limits, if exist. (8%)
 - (a) $\lim_{n \rightarrow \infty} \left(\sqrt{1 - \frac{3}{n}} \right)^n$
 - (b) $\lim_{k \rightarrow \infty} \left(\frac{k}{k+1} \right)^k$
 - (c) $\lim_{t \rightarrow 0} \frac{\sin 2t^3}{t^3}$
 - (d) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$
3. Find the following values, if exist. (20%)
 - (a) $\int_{-2}^2 3 + \sqrt{4 - x^2} dx$
 - (b) $\int_{\sqrt{e}}^{e^3} \frac{(\ln x)^5}{x} dx$
 - (c) $\int_0^{\infty} x^3 e^{-x} dx$
 - (d) $\int_{-3}^4 |x^2 - 4| dx$
4. Find the slope of tangent line to the graph of f at the point $(e, f(e))$, where $f(x) = \int_1^{x^2} (\ln t)^2 dt$. (6%)
5. Let $y = \frac{(6x - 5)^4 (4x - 3)^3}{\sqrt{x^2 + 7} (x + 3)^2}$. Approximate the value of y for $x = 1.02$. (10%)
6. Find the average value of $f(x) = \frac{x^3}{\sqrt{27x^4 + 9}}$ over the interval $[0, 1]$. (10%)
7. Use definite integral to derive that the volume of a cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$. (10%)
8. Find the value of the following infinite series. (10%)
 - (a) $\sum_{n=2}^{\infty} n(n-1)0.5^n$
 - (b) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$
9. Find the local extrema and saddle points of (10%)

$$f(x, y) = \frac{1}{3}x^3 + \frac{3}{4}y^3 - x^2 - 3x - 4y - 3.$$
10. Evaluate the double integral $\int \int_R x^3 \cos xy dx dy$, where the region R is bounded by $y = x^2$ and $y = 0$, and $x = 2$. (10%)