

注意：(1)請按題號作答；(2)答案務必要寫明演算過程且化簡，否則不予計分。

1. (10分) A triangle has two sides of length 1 and the angle between these two sides is θ . Express the area of the triangle as a function of θ . Then find the maximum possible area of such a triangle using calculus.

2. (10分) Find d^2y/dx^2 given $\sqrt{x} + \sqrt{y} = 1$. Simplify your answer as much as you can.

3. (10分) The first-quadrant region bounded below by the x -axis and above by the graph of $y = x^3 - 3x^2 + 2x$ is rotated around the y -axis. Find the volume of the solid thereby generated.

4. Find $f'(x)$ given $f(x) =$ (20分)

- (a) x^{2n} (b) $x^{\ln x}$
(c) $\log_{10} x$ (d) $(\sin x)^n$

5. (20分) Find the indicated antiderivatives and definite integrals:

(a) $\int 6^x dx$ (b) $\int \frac{1}{x} 10^{10x} dx$

(c) $\int_0^1 x \cdot 2^{-x^2} dx$ (d) $\int_1^3 5^{-x} dx$

6. (10分)

- (a) Evaluate $\lim_{x \rightarrow \infty} \left(\frac{x-1}{x} \right)^x$. (b) Evaluate $\lim_{x \rightarrow \infty} (\ln x)^{1/x}$.

7. (10分) Derive and write the Taylor series with center $c = 0$ for the function $f(x) = \cosh x$. Find the interval of convergence of this series.

8. (10分) The plane region R is bounded by the graphs of $y = x$ and $y = x^2$. Find the volume over R and beneath the graph of $f(x, y) = x + y$.