1 Consider a random variable X with a probability density function described by

$$f(x) = 0.2 + kx, \text{ for } -5 < x \leq 0$$
$$= 0.2 - kx, \text{ for } 0 < x < 5$$

where k is some constant.

(5%) a. Find k and verify that f(x) is a probability density function.

(5%) b. Compute Var(X).

(5%) c. Compute the 90th percentile.

(5%) d. Compare the X variable falling into the intervals $(\mu-2\sigma, \mu+2\sigma)$ with the proportions suggested by the Empirical Rule and Chebyshev's theorem.

2 A snow-removal company bills its customers on a per-snowfall basis, rather than at a flat monthly rate. Based on the fee it charges per snowfall, the company will just break even in a month that has exactly six snowfalls. Suppose that the average number of snowfalls per month (during the winter) is eight.

(5%) a. What is the probability that the company will just break even in a given winter month?

(5%) b. What is the probability that the company will make a profit in a given winter month?

(背面仍有題目,請繼續作答)

(15%) 3 Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed normal distribution with mean μ and variance σ^2. Prove that an estimator $m = \sum w_i x_i$ is the most efficient and unbiased estimator when $w_i = (1/n)$.

4 The owner of a downtown parking lot suspects that the person she hired to run the lot is stealing some money. The receipts as provided by the employee indicate that the average number of cars parked in the lot is 150 per day and that, on average, each car is parked for 3 hours. In order to determine whether the employee is stealing, the owner watches the lot for 5 days. On those days, the number of cars parked is as follows:

140, 160, 155, 170, 145

For the 770 cars that the owner observed during the 5 days, the mean and the standard deviation of the time spent on the lot were 3.2 hours and 0.4 hour, respectively.

(5%) a. Can the owner conclude at the 5% level of significance that the employee is stealing?

(5%) b. Discuss the consequence of Type I and Type II errors.

(5%) c. If you are the owner, do you want a small or large value of Type I error? Explain.

(5%) d. If you are the employee, do you want a small or large value of Type I error? Explain.

5 Consider the following standard bivariate regression model:

$$Y_t = \alpha + \beta X_t + u_t, \quad t = 1, 2, \ldots, T.$$

(10%) a. Define B.L.U.E. and explain the statistical or mathematical meanings of B, L, and U. Under what assumptions and conditions are OLS estimates B.L.U.E.?

(5%) b. Prove that $V(\hat{\beta}) = \sigma^2/[\sum(X_t-\bar{X})^2]$.

(10%) c. Consider an alternative estimator $\beta^* = (Y_T-Y_1)/(X_T-X_1)$. Is β^* unbiased? Find $V(\beta^*)$ and compare it with $V(\hat{\beta})$. Is β^* more efficient than $\hat{\beta}$?

(10%) d. Define R^2 and explain why it is used to measure the goodness of fit of a regression model. What is the main shortcoming of R^2? How would you correct for the shortcoming?

參考數值表：

<1> $\sqrt{2}=1.414$, $\sqrt{3}=1.732$, $\sqrt{5}=2.236$, $\sqrt{6}=2.449$

<2>

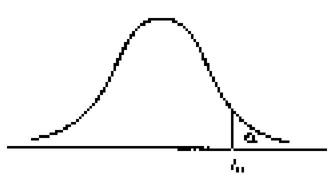
TABLE 2 Poisson Probabilities

Tabulated values are $P(x \le k) = \sum_{x=0}^{k} p(x)$. (Values are rounded to three decimal places.)

k	μ = 6.5	7.0	7.5	8.0	8.5	9.0	9.5	10	11	12	13	14	15
0	.002	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.011	.007	.005	.003	.002	.001	.001	.000	.000	.000	.000	.000	.000
2	.043	.030	.020	.014	.009	.006	.004	.003	.001	.001	.000	.000	.000
3	.112	.082	.059	.042	.030	.021	.015	.010	.005	.002	.001	.000	.000
4	.224	.173	.132	.100	.074	.055	.040	.029	.015	.008	.004	.002	.001
5	.369	.301	.241	.191	.150	.116	.089	.067	.038	.020	.011	.006	.003
6	.527	.450	.378	.313	.256	.207	.165	.130	.079	.046	.026	.014	.008
7	.673	.599	.525	.453	.386	.324	.269	.220	.143	.090	.054	.032	.018
8	.792	.729	.662	.593	.523	.456	.392	.333	.232	.155	.100	.062	.037
9	.877	.830	.776	.717	.653	.587	.522	.458	.341	.242	.166	.109	.070
10	.933	.901	.862	.816	.763	.706	.645	.583	.460	.347	.252	.176	.118

<3>

TABLE 3 Critical Values of t



DEGREES OF FREEDOM	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	DEGREES OF FREEDOM	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657	24	1.318	1.711	2.064	2.492	2.797
2	1.886	2.920	4.303	6.965	9.925	25	1.316	1.708	2.060	2.485	2.787
3	1.638	2.353	3.182	4.541	5.841	26	1.315	1.706	2.056	2.479	2.779
4	1.533	2.132	2.776	3.747	4.604	27	1.314	1.703	2.052	2.473	2.771
5	1.476	2.015	2.571	3.365	4.032	28	1.313	1.701	2.048	2.467	2.763
6	1.440	1.943	2.447	3.143	3.707	29	1.311	1.699	2.045	2.462	2.756
7	1.415	1.895	2.365	2.998	3.499	30	1.310	1.697	2.042	2.457	2.750
8	1.397	1.860	2.306	2.896	3.355	35	1.306	1.690	2.030	2.438	2.724
9	1.383	1.833	2.262	2.821	3.250	40	1.303	1.684	2.021	2.423	2.704
10	1.372	1.812	2.228	2.764	3.169	45	1.301	1.679	2.014	2.412	2.690
11	1.363	1.796	2.201	2.718	3.106	50	1.299	1.676	2.009	2.403	2.678
12	1.356	1.782	2.179	2.681	3.055	60	1.296	1.671	2.000	2.390	2.660
13	1.350	1.771	2.160	2.650	3.012	70	1.294	1.667	1.994	2.381	2.648
14	1.345	1.761	2.145	2.624	2.977	80	1.292	1.664	1.990	2.374	2.639
15	1.341	1.753	2.131	2.602	2.947	90	1.291	1.662	1.987	2.368	2.632
16	1.337	1.746	2.120	2.583	2.921	100	1.290	1.660	1.984	2.364	2.626
17	1.333	1.740	2.110	2.567	2.898	120	1.289	1.658	1.980	2.358	2.617
18	1.330	1.734	2.101	2.552	2.878	140	1.288	1.656	1.977	2.353	2.611
19	1.328	1.729	2.093	2.539	2.861	160	1.287	1.654	1.975	2.350	2.607
20	1.325	1.725	2.086	2.528	2.845	180	1.286	1.653	1.973	2.347	2.603
21	1.323	1.721	2.080	2.518	2.831	200	1.286	1.653	1.972	2.345	2.601
22	1.321	1.717	2.074	2.508	2.819	∞	1.282	1.645	1.960	2.326	2.576
23	1.319	1.714	2.069	2.500	2.807						

Source: From M. Merrington, "Table of Percentage Points of the t-Distribution," Biometrika 32 (1941): 300. Reproduced by permission of the Biometrika Trustees.