微積分

1. 已知 $y^2 - 6px = a, (p > 0)$, 求 $\dfrac{dy}{dx}$ 及通過 (x_0, y_0) 之切線方程式。(10%)

2. $x = a\cos t$，$y = b\sin t$。 求 1) dy/dx;   2) 當 $t = \pi/2$ 時，$dy/dx = ?$。(10%)

3. 設 $\int f(x)dx = F(x)$. 試證 1) $\int f(x-a)dx = F(x-a)$，$a \in R$。

   2) $\int f(x\gamma)dx = F(x\gamma)/\gamma$，$\gamma \neq 0$。(10%)

4. 設 $f(x, y) = x^2 + xy - y^2$。試求 $f(x, y)$ 在 $(1, -2)$ 的泰勒 (Taylor) 展開式。

   (10%)

5. Evaluate $\int_0^3 x\sqrt{1+x}\,dx$. (10%)

6. A function $f: R \to R$ is defined by

   $$f(x) = \begin{cases} -x^2 & \text{when } x < 0 \text{ and} \\ x^2 & \text{when } x \geq 0. \end{cases}$$

   Determine whether 1) f is continous at 0, 2) $f'(0)$ exists. (10%)

7. Let $\{x_n\}$ $(n = 1, 2, ...)$ be an increasing sequence, and assume that it is bounded from above.   Show that the least upper bound b of the set $\{x_n\}$ $(n = 1, 2, ...)$ is the limit of the sequence.   (20%)

8. Show that $\lim\limits_{n \to \infty} \dfrac{2n}{n-1} = 2$. (20%)