統計學

1. Suppose that X and Y are independent random variables. Let $Z = X + Y$. Let $M_X(t)$, $M_Y(t)$, and $M_Z(t)$ be the moment-generating functions of X, Y, and Z. Show that $M_Z(t) = M_X(t)\, M_Y(t)$. (10%)
2. Show that E(E(X|Y)) = E(X). (10%)
3. Let X be uniformly distributed on [-1, 1]. Find the density of $Y = X^r$ for non-negative integers r. (20%)
4. Show that for $\varepsilon > 0$, as $n \to \infty$, then $P\left(\left|\frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)\right| > \varepsilon\right) \to 0$. (15%)
5. Let X be a random variable with $E(X) = \mu$, and $V(X) = \sigma^2$. Suppose that $Y = H(X)$. Show that 1)$E(Y) \cong H(\mu) + \frac{H''(\mu)}{2}\sigma^2$; 2)$V(Y) \cong [H'(\mu)]^2\sigma^2$. $E(.)$ and $V(.)$ denote expectation and variance. (20%)
6. Suppose that the joint pdf of (X, Y) is given by

$$f(x,y) = \begin{cases} e^{-y}, & \text{for } x > 0, y > x, \\ 0, & \text{elsewhere.} \end{cases}$$

1)Find the marginal pdf of X.
2)Find the marginal pdf of Y. (15%)

7. Suppose that a continuous random variable has cdf F given by

$$F(x) = \begin{cases} 0, & x \le 0, \\ 1 - e^{-x}, & x > 0. \end{cases}$$

Find the pdf. (10%)