

一. 選擇題: 60% (每題 6 分)

1. If two functions f and g are continuous at a real number c , then which function is also continuous at c : (a) the sum $f + g$ (b) the difference $f - g$ (c) the product $f \cdot g$ (d) the quotient $\frac{f}{g}$, provide $g(c) \neq 0$ (e) all of them

2. The hyperbolic sine function is denoted by \sinh . Find the derivative of the inverse function of it. That is, let $y = \sinh^{-1} x$, find $\frac{dy}{dx} =$ (a) $\frac{1}{\sqrt{x^2+1}}$ (b) $\frac{x}{\sqrt{x^2+1}}$

(c) $\frac{1}{\sqrt{x^2+x}}$ (d) $\frac{x^2}{\sqrt{x^2+1}}$ (e) $\frac{x}{\sqrt{x^2+x}}$

3. Evaluate $\lim_{M \rightarrow \infty} \int_0^M \frac{dx}{x^4+4} = c$, c is a constant. What is the value of c ? (a) $\frac{\pi}{2}$ (b)

$\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$ (e) $\frac{\pi}{12}$

4. What is the value of $\int_0^1 \frac{1-e^{-x^2}}{x^2} dx$? (精確到第三位) (a) 0.710 (b) 0.637 (c) 0.231 (d) 0.425 (e) 0.862

5. Find the limits of $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$ (a) 1 (b) 0 (c) ∞ (d) $-\infty$ (e) none

6. $\varepsilon = 0.01$, find a minimum positive integer N . For all $n > N$, $\left| \frac{3n+2}{n-1} - 3 \right| < \varepsilon$,
 $N =$ (a) 302 (b) 402 (c) 502 (d) 602 (e) 702

7. $u_{n+1} = \sqrt{u_n+1}$, $u_1 = 1$. The value of $\lim_{n \rightarrow \infty} u_n =$ (a) $1+\sqrt{5}$ (b) $\frac{1}{2}(1+\sqrt{3})$

(c) $\frac{1}{2}(1+\sqrt{5})$ (d) $\frac{1}{2}(\sqrt{3}+\sqrt{5})$ (e) $\frac{1}{3}(2+\sqrt{5})$

(背面仍有題目, 請繼續作答)

8. $a = \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$, $b = \lim_{x \rightarrow 0^+} \left(\frac{1}{e^x - 1} - \frac{1}{x} \right)$. What is the value of (a, b) (a)

$\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (c) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (e) $\left(1, \frac{1}{2}\right)$

9. Let $y = x \arcsin(x)$, $z = \ln|4 + 5x - 2x^3|$. Find $\frac{dy}{dx}$ and $\frac{dz}{dx}$

(a) $\arcsin(x) + \frac{1}{\sqrt{1-x^2}} + C_1, \frac{5-6x^2}{4+5x-2x^3} + C_2$

(b) $\arcsin(x) + \frac{1}{\sqrt{1-x^2}} + C_1, \frac{|5-6x^2|}{4+5x-2x^3} + C_2$

(c) $\arcsin(x) + \frac{x}{\sqrt{1-x^2}} + C_1, \frac{5-6x^2}{|4+5x-2x^3|} + C_2$

(d) $\arcsin(x) + \frac{1}{\sqrt{1-x^2}} + C_1, \left| \frac{5-6x^2}{4+5x-2x^3} \right| + C_2$

(e) $\arcsin(x) + \frac{x}{\sqrt{1-x^2}} + C_1, \frac{5-6x^2}{4+5x-2x^3} + C_2$ (C_1 and C_2 are constants)

10. V is the volume of the region beneath the surface $z = xy^2 + y^3$ and over the rectangle $R = \{(x, y) : 0 \leq x \leq 2 \text{ and } 1 \leq y \leq 3\}$. What is the value of V ?

(a) $\frac{152}{3}$ (b) $\frac{161}{3}$ (c) $\frac{172}{3}$ (d) $\frac{292}{3}$ (e) $\frac{224}{3}$

二. 計算證明題:40%

1. If $y = x^x$ and $x > 0$, Find $\frac{dy}{dx}$ (10%)

2. Evaluate the following functions:

a. $\int \left(\frac{5x^3 - 3x^2 + 7x - 3}{(x^2 + 1)^2} + x \cdot 3^{x^2} \right) dx$ (5%)

b. $\int (x^2 e^{2x} + e^x \cos x) dx$ (5%)

3. Determine whether the series is convergent or divergent ?

a. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ (5%)

b. $\sum_{n=1}^{\infty} \frac{2^{3n+1}}{n^n}$ (5%)

4. Pick a_0 and a_1 . For $n \geq 2$, compute a_n recursively, so that

$$n(n-1)a_n = (n-1)(n-2)a_{n-1} - (n-3)a_{n-2}$$

Evaluate $\sum_{n=0}^{\infty} a_n$. (10%)