※本考題附有常態分配及 t 分配表

選擇題: (50%)

1. (5%) If X is a random variable with mean 33 and variance 16, what is the lower bound for $P(23 < X < 43)$ and upper bound for $P(|X-33| \geq 14)$.(Hint: Use Chebyshev's inequality to find the answer) (a) 0.54, 0.082 (b) 0.84 , 0.082 (c) 0.62, 0.018 (d) 0.16 , 0.018 (e) 0.54, 0.38

2. (5%) If 10 fair dice are rolled, find the approximate probability that the sum obtained is between 30 and 40. (Hint: Use central limit theorem to find the answer) (a) 0.21 (b) 0.32 (c) 0.46 (d) 0.65 (e) 0.84

3. (5%) Assume that IQ scores for a certain population are approximately $N(\mu,100)$. To test $H_0: \mu = 110$ against the one-sided alternative hypothesis $H_1: \mu > 110$, we take a random sample of size $n = 16$ from this population and observe $\bar{x} = 113.5$. Do we accept or reject H_0 at 5% significance level ? What is the p-value of this test? (a) Reject H_0, p-value=0.0808 (b) Do not reject H_0, p-value=0.0808 (c) Reject H_0, p-value=0.0533 (d) Do not reject H_0, p-value=0.0533 (e) Reject H_0, p-value=0.0418.

4. (5%) A 5-card poker hand is said to be a full house if it consists of 3 cards of the same denomination and 2 cards of the same denomination. (that is, a full house is three of a kind plus a pair.) What is the probability of being dealt a full house? (a)0.025 (b) 0.0033 (c) 0.0014 (d) 0.0088 (e) 0.0020

5. (5%) What is the distribution that has the same mean and variance ? (a) Normal Distribution (b) T Distribution (c) Poisson Distribution (d) Geometric Distribution (e) none

6. (5%) Let $Y_1 < Y_2 < \cdots < Y_{19}$ be the order statistics of a random sample of size $n = 19$ From the exponential distribution with mean θ. What is the p.d.f of Y_1? What is the value of $E[F(Y_1)]$, where F is the distribution function of the exponential distribution. (a) $g_1(y) = 18(e^{-y/\theta})^{19}\frac{1}{\theta}e^{-y/\theta}$ where $0 < y < \infty$, 1/19 (b) $g_1(y) = 19(e^{-y/\theta})^{19}\frac{1}{\theta}e^{-2y/\theta}$ where $0 < y < \infty$, 1/19 (c) $g_1(y) = 19(e^{-y/\theta})^{18}\frac{1}{\theta}e^{-y/\theta}$

(背面仍有題目，請繼續作答)

where $0 < y < \infty$, 1/20 (d) $g_1(y) = 19(e^{-2y/\theta})^{18}\frac{1}{\theta}e^{-2y/\theta}$ where $0 < y < \infty$, 1/20

7. (5%) Suppose we have 3 cards identical in form except that both sides of the first card are colored red, both sides of the second card are colored blacked, and one side of the third card is colored red and the other side black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black? (a) 1/3 (b)1/2 (c) 2/3 (d) 1/4 (e) 1/6

8. (5%) Let X and Y equal the lengths in inches of a foot and a hand, respectively. The following measurements were made on 15 woman.

x	y	x	y	x	y
9.00	6.50	10.00	7.00	9.25	7.00
8.50	6.25	9.50	6.50	10.00	7.50
9.25	7.25	9.00	7.00	10.00	7.25
9.75	7.00	9.25	7.00	9.75	7.25
9.00	6.75	9.50	7.00	9.50	7.25

What is the least squares regression line for these data?
(a) $\hat{y} = 1.896 + 0.538x$ (b) $\hat{y} = 0.538 + 1.896x$ (c) $\hat{y} = 0.896 + 1.538x$ (d) $\hat{y} = 0.538 - 1.896x$ (e) $\hat{y} = 1.896 - 0.538x$

9. (5%) Thirteen tons of cheese is stored in some old gypsum mines, including "22-pound" wheels (label weight). A random sample of $n = 9$ of these wheels yielded the following weights in pounds:

21.5 18.95 18.55 19.40 19.15
22.35 22.90 22.20 23.10

Assuming that the distribution of the weights of the wheels of cheese is $N(\mu, \sigma^2)$, which is the following answer is a 95% confidence interval for μ?
(a) [25.13,28.43](b)[12.34,15.33] (c)[33.23, 35.72] (d)[24.23, 27.65] (e)[19.47, 22.33]

10. (5%) Let $f(x) = xe^x, 0 < x < 1$ be the p.d.f of X. What is the value of μ (mean) and σ^2 (variance) (a) $e-1$, $e+2-e^2$ (b) $1-e$, e^2-e+2 (c) $e-2$, $2+2e-e^2$ (d) $2-e$, e^2+2e+2 (e) $e-2$, e^2+2e+2

計算證明題: (50%)

1. (10%) The correlation of two random variables X and Y, denoted by $\rho(X,Y)$, is defined, as long as $Var(X)$ and $Var(Y)$ is positive, by

$$\rho(X,Y)=\frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

Show that $-1\le\rho(X,Y)\le 1$.

2. (10%) Show that $Var(x)=E[Var(X|Y)]+Var(E[X|Y])$.

3. (10%) X is a random variable. Show that if $f(x)$ is a convex function, then

$$E[f(X)]\ge f(E[X])$$

provided that the expectations exist and are finite.

4. (10%) If the moment-generating function of X is

$$M(t)=\frac{2}{5}e^{t}+\frac{1}{5}e^{2t}+\frac{2}{5}e^{3t},$$

find the mean, variance, p.d.f. of X, and the probability-generating function.

5. (10%) Let $X_1,X_2,.....,X_n$ be a random sample from a normal distribution.
(a)(6%) show that an unbiased estimator for σ is cS, where

$$c=\frac{\sqrt{n-1}\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{2}\Gamma\left(\frac{n}{2}\right)}.\quad(\Gamma(t)=\int_0^\infty e^{-y}y^{t-1}dy.)$$

(b) (4%)find the value of c when $n=5,n=6$.

(背面仍有題目,請繼續作答)

國立成功大學九十四學年度碩士班招生考試試題

編號：361 系所：會計學系乙組 科目：統計學

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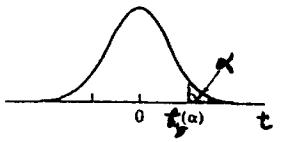
TABLE 1 STANDARD NORMAL PROBABILITIES

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998

TABLE 2 STUDENT'S t-DISTRIBUTION CRITICAL POINTS



d.f. ν	α .250	.100	.050	.025	.010	.00833	.00625	.005
1	1.000	3.078	6.314	12.706	31.821	38.190	50.923	63.657
2	.816	1.886	2.920	4.303	6.965	7.649	8.860	9.925
3	.765	1.638	2.353	3.182	4.541	4.857	5.392	5.841
4	.741	1.533	2.132	2.776	3.747	3.961	4.315	4.604
5	.727	1.476	2.015	2.571	3.365	3.534	3.810	4.032
6	.718	1.440	1.943	2.447	3.143	3.287	3.521	3.707
7	.711	1.415	1.895	2.365	2.998	3.128	3.335	3.499
8	.706	1.397	1.860	2.306	2.896	3.016	3.206	3.355
9	.703	1.383	1.833	2.262	2.821	2.933	3.111	3.250
10	.700	1.372	1.812	2.228	2.764	2.870	3.038	3.169
11	.697	1.363	1.796	2.201	2.718	2.820	2.981	3.106
12	.695	1.356	1.782	2.179	2.681	2.779	2.934	3.055
13	.694	1.350	1.771	2.160	2.650	2.746	2.896	3.012
14	.692	1.345	1.761	2.145	2.624	2.718	2.864	2.977
15	.691	1.341	1.753	2.131	2.602	2.694	2.837	2.947
16	.690	1.337	1.746	2.120	2.583	2.673	2.813	2.921
17	.689	1.333	1.740	2.110	2.567	2.655	2.793	2.898
18	.688	1.330	1.734	2.101	2.552	2.639	2.775	2.878
19	.688	1.328	1.729	2.093	2.539	2.625	2.759	2.861
20	.687	1.325	1.725	2.086	2.528	2.613	2.744	2.845
21	.686	1.323	1.721	2.080	2.518	2.601	2.732	2.831
22	.686	1.321	1.717	2.074	2.508	2.591	2.720	2.819
23	.685	1.319	1.714	2.069	2.500	2.582	2.710	2.807
24	.685	1.318	1.711	2.064	2.492	2.574	2.700	2.797
25	.684	1.316	1.708	2.060	2.485	2.566	2.692	2.787
26	.684	1.315	1.706	2.056	2.479	2.559	2.684	2.779
27	.684	1.314	1.703	2.052	2.473	2.552	2.676	2.771
28	.683	1.313	1.701	2.048	2.467	2.546	2.669	2.763
29	.683	1.311	1.699	2.045	2.462	2.541	2.663	2.756
30	.683	1.310	1.697	2.042	2.457	2.536	2.657	2.750
40	.681	1.303	1.684	2.021	2.423	2.499	2.616	2.704
60	.679	1.296	1.671	2.000	2.390	2.463	2.575	2.660
120	.677	1.289	1.658	1.980	2.358	2.428	2.536	2.617
∞	.674	1.282	1.645	1.960	2.326	2.394	2.498	2.576