編號：G 355　系所：會計學系乙組　　科目：微積分

本試題是否可以使用計算機：□可使用，☑不可使用　（請命題老師勾選）

一、選擇題：50% (每題 5 分)

1. Find the value $\lim_{n\to\infty}\left[\frac{(3n)!}{n!n^{2n}}\right]^{\frac{1}{n}}$ = (a) $\frac{5}{e^3}$ (b) $\frac{27}{e}$ (c) $\frac{5}{e}$ (d) $\frac{27}{e^2}$ (e) $\frac{5}{e^2}$

2. Evaluate $\lim_{n\to\infty}\frac{2}{1}\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\cdots\frac{2n}{2n-1}\frac{2n}{2n+1}$ = (a) 1 (b) ∞ (c) 0 (d) $\frac{\pi}{2}$ (e) $\frac{\pi}{4}$

3. Find the value $\int_0^3|x-1|dx$ = (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{5}{2}$ (e) 6

4. Find the area of the region between the graphs of the equations $y=x-2$ and $y=2x-x^2$ (a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{9}{2}$ (e) $\frac{11}{2}$

5. Find the value $\int\frac{x}{\sqrt{7+2x-x^2}}dx$ = (a) $-\sqrt{7+2x-x^2}+\sin^{-1}\frac{x-1}{\sqrt{8}}+C$

(b) $\sqrt{7+2x-x^2}+\sin^{-1}\frac{x-1}{\sqrt{8}}+C$ (c) $-\frac{1}{2}\sqrt{7+2x-x^2}+2\sin^{-1}\frac{x-1}{\sqrt{8}}+C$

(d) $-\sqrt{7+2x-x^2}+\frac{1}{2}\sin^{-1}\frac{x-1}{\sqrt{8}}+C$ (e) $-\frac{1}{2}\sqrt{7+2x-x^2}+\sin^{-1}\frac{x-1}{\sqrt{8}}+C$

6. Evaluate $\lim_{\theta\to\frac{\pi}{2}^-}\frac{\sec\theta}{\tan\theta}$ = (a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) ∞ (e) 1

7. Evaluate $\int\sqrt{x^2+a^2}dx$ = (a) $\frac{a^2}{2}\sinh^{-1}\frac{x}{a}+\frac{1}{2}\sqrt{x^2+a^2}$

(b) $\frac{a^2}{2}\sinh^{-1}\frac{x}{a}+\frac{x^2}{2}\sqrt{x^2+a^2}$ (c) $\frac{a^2}{2}\sinh^{-1}\frac{x}{a}+\frac{x}{2}\sqrt{x^2+a^2}$

(d) $\frac{a^2}{2}x\sinh^{-1}\frac{x}{a}+\frac{x}{2}\sqrt{x^2+a^2}$ (e) $\frac{a^2}{2}\sinh^{-1}\frac{x^2}{a^2}+\frac{x}{2}\sqrt{x^2+a^2}$

8. Let $y=x^x$ and $x>0$, find $\frac{d^2y}{dx^2}$ =

(a) $x^{x-1}(1+x(1+\log x)^2)$ (b) $x^x(1+x(1+\log x)^2)$ (c) $x(1+x^{x-1}(1+\log x)^2)$

(d) $x^{x-1}(1+\log(x-1)(1+\log x)^2)$ (e) $x^{x-1}(1+x(1+\log x))$

9. Evaluate $\int\frac{x^2+4x+4}{(x+1)^2(x^2+3x+3)}dx$ =

(a) $-\frac{1}{x+1}+\log|x+1|-\frac{1}{2}\log|x^2+3x+3|+\frac{1}{2\sqrt{3}}\tan^{-1}\frac{(2x+3)}{\sqrt{3}}+C$

(背面仍有題目,請繼續作答)

(b) $-\frac{1}{x+1}+\log|x+1|+\frac{1}{3}\log|x^2+3x+3|-\frac{1}{\sqrt{3}}\tan^{-1}\frac{(2x+3)}{\sqrt{3}}+C$

(c) $-\frac{1}{x+1}-\log|x+1|+\frac{1}{3}\log|x^2+3x+3|+\frac{1}{2\sqrt{3}}\tan^{-1}\frac{(2x+3)}{\sqrt{3}}+C$

(d) $-\frac{1}{x+1}+\log|x+1|-\frac{1}{2}\log|x^2+3x+3|-\frac{1}{\sqrt{3}}\tan^{-1}\frac{(2x+3)}{\sqrt{3}}+C$

(e) $-\frac{1}{x+1}+\log|x+1|-\frac{1}{2}\log|x^2+3x+3|-\frac{2}{\sqrt{3}}\tan^{-1}\frac{(2x+3)}{\sqrt{3}}+C$

10. Find the value of $\lim_{\alpha\to 0}\frac{\sin\alpha}{\alpha}$, where α is measured "in degrees" (a) 1 (b) 0 (c) ∞

(d) π (e) $\frac{\pi}{180}$

二、計算證明題：50%

1. Show that $\lim_{x\to 0^+} x^x = 1$. (10%)

2. Prove that if the integer $n>1$

$$\int \sec^n x\,dx = \frac{1}{n-1}\left[\sec^{n-2}x\tan x+(n-2)\int \sec^{n-2}x\,dx\right] \quad (10\%)$$

3. Given the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with circumference L, show that

$$L = 4a\int_0^{\frac{\pi}{2}}\sqrt{1-e^2\sin^2 u}\,du$$

where $e=\frac{\sqrt{a^2-b^2}}{a}$ (e is called the eccentricity of the ellipse.) (10%)

4. Use spherical coordinates to compute the triple integral $\iiint_S f$, if

$$f(x,y,z)=\frac{z^2}{\sqrt{x^2+y^2+z^2}}$$

and S is the region between the sphere of radius a and b centered at the origin $(0<a<b)$ and above the xy-plane. (10%)

5. Show that the approximation

$$\int_a^b F(x)dx = \frac{b-a}{6}\left[F(a)+4F\left(\frac{a+b}{2}\right)+F(b)\right],$$

which is Simpson's rule for $n=2$, is exact for $F(x)$ a cubic polynomial. (10%)