本試題是否可以使用計算機：☑可使用 ，□不可使用 （請命題老師勾選）

一、選擇題 50分(每題五分)

1. Evaluate $\iint_R f(x,y)dA$, where $f(x,y)=x+2y$ and R is the rectangle defined by $1\le x\le 4$ and $1\le y\le 2$. (a) $\frac{15}{2}$ (b) 6 (c) $8\frac{1}{2}$ (d) $16\frac{1}{2}$

2. Suppose that x units of labor and y units of capital are required to produce $f(x,y)=100x^{3/4}y^{1/4}$ units of a certain product (recall that this is a Cobb-Douglas production function). If each unit of labor costs \$200 and each unit of capital costs \$300, and a total of \$60,000 is available for production, determine how many units of capital should be used in used in order to maximize production.

   (a) $x=150, y=100$ (b) $x=200, y=\frac{200}{3}$ (c) $x=225, y=50$ (d) $x=250, y=\frac{100}{3}$

3. Evaluate $\int\frac{(\ln x)^2}{2x}dx$ (a) $\frac{1}{6}x(\ln x)^2+c$ (b) $\frac{1}{6}x(\ln x)^3+c$ (c) $\frac{1}{6}x(\ln x^2)+c$ (d) $\frac{1}{6}(\ln x)^3+c$

4. Find $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=$ (a) 0 (b) 1 (c) ∞ (d) e

5. Find $\lim_{x\to\infty}(x^2+a^2)^{(\frac{1}{x})^2}=$ (a) 0 (b) 1 (c) ∞ (d) e

6. Find an equation of the tangent line to the graph of $y=\sqrt{4-x^2}$ at the point $(1,\sqrt{3})$. (a) $y=-\sqrt{3}x+2\sqrt{3}$ (b) $y=-2\sqrt{3}x+3\sqrt{3}$ (c) $y=-\frac{\sqrt{3}}{3}x+\frac{4}{3}\sqrt{3}$ (d) $y=\frac{\sqrt{3}}{3}x+\frac{2}{3}\sqrt{3}$

7. Evaluate $\int_{-\infty}^{\infty}xe^{-x^2}dx=$ (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

8. Find the area between $y=\cos x$ and $y=\sin x$, $x\in[0,2\pi]$ (a) 2 (b) $2\sqrt{2}$ (c) 4 (d) $4\sqrt{2}$

9. Find $\int\frac{\sec x}{2\tan x+\sec x-1}dx=$(a) $\log\left|\frac{\tan\frac{1}{2}x}{\tan\frac{1}{2}x+1}\right|+C$ (b) $\frac{1}{2}\log\left|\frac{\tan x}{\tan x+2}\right|+C$ (c)

（背面仍有題目，請繼續作答）

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$\frac{1}{2}\log\left|\frac{\tan\frac{1}{2}x}{\tan\frac{1}{2}x+2}\right|+C$ (d) $\frac{1}{2}\log\left|\frac{\tan x}{\tan\frac{1}{2}x+2}\right|+C$

10. Evaluate $\int_0^e\int_{-2x}^{1-x}\int_0^{\sqrt{2-x^2}} z\,dz\,dy\,dx$ (a) 0 (b) -1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

二、非選擇題 50 分

1. (20%) Evaluate the following functions:

a. $\int x^2 e^{-\frac{1}{2}x}dx$ (5%)

b. $\int(\frac{x^5+2}{x^2-1})dx$ (5%)

c. $\int x^3 e^x dx$ (5%)

d. $\int\tan^6 x dx$ (5%)

2. (10%) Prove that for each positive integer n,

$$\left(1+\frac{1}{n}\right)^n \le e \le \left(1+\frac{1}{n}\right)^{n+1}$$

3. (10%) Estimate

$$\int_0^e \sqrt{4+x^3}\,dx$$

(a) (5%) by the trapezoidal rule. Take $n=4$.

(b) (5%) by the Simpson's rule. Take $n=2$.

4. (10%) Show that

$$\int_{-\infty}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}dx=1$$

for any μ and for $\sigma>0$.