一、選擇題 50 分(每題五分)

1. $\sin^{-1}x+\cos^{-1}x=$ (a) $\frac{\sqrt{\pi}}{2}$ (b) π (c) $\frac{2}{\pi}$ (d) $\frac{\pi}{2}$

2. The area of the curve $y=\sqrt{1-x^2}$ from $x=-1$ to $x=1$ is rotated about the X-axis. Find the resulting surface area (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) 2π (d) 4π

3. $\int_2^{2\cosh x}\frac{dt}{\sqrt{t^2-4}}=$ (a) $\sinh x$ (b) $\sin^{-1}x$ (c) x (d) x^2

4. If $x>0$, $\lim_{n\to\infty}\sqrt[n]{x}=$ (a) ∞ (b) 1 (c) 0 (d) x

5. $\lim_{n\to\infty}\left(\sqrt{n^2+n}-n\right)=$ (a) ∞ (b) $-\infty$ (c) 0 (d) $\frac{1}{2}$

6. $\int\frac{x^5-x^4-2x^3+4x^2-15x+5}{(x^2+1)^2(x^2+4)}dx=$

(a) $\frac{1}{2}\ln\left|x^2+4\right|-\frac{3}{2}\tan^{-1}\frac{x}{2}+2\tan^{-1}x+\frac{2}{x^2+1}+C$

(b) $\frac{1}{2}\ln\left|x^2+4\right|+\frac{3}{2}\tan^{-1}2x+2\tan^{-1}x+\frac{2}{x^2+1}+C$

(c) $\frac{1}{2}\ln\left|x^2+4\right|+\frac{3}{2}\tan^{-1}\frac{x}{2}+2\tan^{-1}x^2+\frac{2}{x^2+1}+C$

(d) $\frac{1}{2}\ln\left|x^2+4\right|-\frac{3}{2}\tan^{-1}2x+2\tan^{-1}x^2+\frac{2}{x^2+1}+C$

7. Evaluate $\int\sin(\ln(x))dx=$ (a) $\frac{x}{2}[\sin(\ln x)+\cos(\ln x)]+C$

(b) $\frac{x}{2}[\sin(\ln x)-\cos(\ln x)]+C$

(c) $\frac{1}{2}[\sin(\ln x)+\cos(\ln x)]+C$

(d) $\frac{1}{2}[\sin(\ln x)-\cos(\ln x)]+C$

8. Find the length of the arc of the cardioid $r=1-\cos\theta$ from $\theta=0$ to $\theta=2\pi$
(a) 2 (b) 4 (c) 8 (d) 12

9. Find $\lim_{x\to1^+}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)=$ (a) 0 (b) ∞ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(背面仍有題目,請繼續作答)

10. Evaluate $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$ (a) 0 (b) -1 (c) $\frac{1}{2}$ (d) ∞

二、非選擇題 50 分

1. (15%) Evaluate the following functions:

   a. $\lim_{x\to 0}(\cos x)^{1/x^2}$ (5%)

   b. $\int \sinh^{-1} x\, dx$ (5%)

   c. Find the derivatives of x^{x^x} (5%)

2. (10%) Prove that $x \geq -1$, and n is a positive integer, then

$$(1+x)^n \geq 1+nx$$

3. (10%) Show that

$$\int_0^{\infty} e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$$

4. (5%) Prove that

$$\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$$ converges, and find its sum.

5. (10%) Prove that

$$\int_0^{\infty} e^{-x}x^n dx = n! \quad (n \in N).$$ N is positive integer.