

※ 考生請注意：本試題可使用計算機

請勿在本試題紙上作答，否則不予計分

一、選擇題：(18分，每題3分)

1. A random sample of 10 items is taken from a normal population. The sample had a mean of 82 and a standard deviation is 26. Which is the appropriate 99% confidence interval for the population mean?
(A) $82 \pm z_{0.005}(26)$ (B) $82 \pm z_{0.0005}(26)$ (C) $82 \pm z_{0.01}(26/\sqrt{10})$ (D) $82 \pm z_{0.005}(26/\sqrt{10})$ (E) none of the above
2. A sample of 150 new cell phones produced by HTC found that 12 had cosmetic flaws. A 90% confidence interval for the proportion of all new HTC phones with cosmetic flaws is 0.044 to 0.116. Which statement below provides the correct interpretation of this confidence interval?
(A) There is a 90% chance that the proportion of new phones that have cosmetic flaws is between 0.044 and 0.116.
(B) If you selected a very large number of samples and constructed a confidence interval for each, 90% of these intervals would include the proportion of all new phones with cosmetic flaws.
(C) There is at least a 4.4% chance that a new phone will have a cosmetic flaw.
(D) A sample of 150 phones will have no more than 11.6% with cosmetic flaws.
(E) none of the above
3. The general partner of a limited partnership firm has told a potential investor that the mean monthly rent for a 3-bedroom home in the area is \$500. The investor wants to check out this claim on her own. She obtains the monthly rental charges for a random sample of 9 three-bedroom homes in order to test $H_0: \mu = 500$ against $H_a: \mu \neq 500$, at $\alpha = 10\%$. The sample mean is \$520 with a sample standard deviation of \$48. Which one is the appropriate rejection region?
(A) $t > 2.306$ (B) $-1.86 < t < 1.86$ (C) $t > 1.833$ (D) $t < -1.86$ or $t > 1.86$ (E) $t < -2.306$ or $t > 2.306$
4. Based on problem 3, what is the value of the test statistic?
(A) 1.25 (B) -1.25 (C) 0.42 (D) -0.42 (E) 20
5. Based on problem 3, in order for the above procedure to be valid, what assumption will be necessary?
(A) The population distribution of the monthly rent is approximately normal.
(B) The population distribution of the monthly rent is uniform.
(C) The population distribution of the monthly rent is skewed.
(D) No assumption will be necessary.
6. Which of the following is TRUE?
(A) Assume events A and B are not disjoint. Then, $P(A \cap B) = P(A) \cdot P(B)$.
(B) The result $E[g(X)] = g[E(X)]$ holds for $g(X) = 2X + 4$, but does not hold for $g(X) = X^2$.
(C) An optimal hypothesis test is defined as the test that minimizes types I and II error.
(D) An interval estimation is just the probability that your estimator (say \bar{x}) is contained in a random interval with probability $1 - \alpha$.
(E) none of above

(背面仍有題目,請繼續作答)

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二、簡答題：（15 分，每題 3 分）

Answer the following questions with an answer and a short explanation. Answers without explanation get at most 1 point.

1. The chance of flipping exactly 50 heads in 100 independent tosses of a fair coin is nearly half. True or False? Explain.
2. When estimating a proportion, a random sample of size 200 from a population of 20,000 is as accurate as a random sample of size 400 from a population of 40,000, assuming the true proportions are the same for both populations. True or False? Explain.
3. Using a t-test instead of a z-test to estimate a mean when a standard deviation is known, the data is roughly normal, and the sample size is small (under 30), will the chance of accepting an alternative hypothesis increase when the null hypothesis is in fact correct. True or False? Explain.
4. If X denotes the number of successes in n independent Bernoulli trials, each trial having success probability p , and if Y denotes the number of failures, what is the variance of $X-Y$ (in terms of n and p)?
5. Based on previous question, what is the variance of $X+Y$?

三、計算題：（67 分）

1. You have a random sample of size n from a $U[0, \theta]$: X_1, \dots, X_n . However, you do not observe X_i directly, but only a signal Y_i of the process. Whenever X_i is greater than 0.5, Y_i takes the value of 1. Whenever X_i is lower or equal than 0.5, Y_i takes the value of 0. (7%)
 - (1) Write down the pdf of Y_i (with value either 0 or 1), denoted by $f(y)$, as a function of θ . (4%)
 - (2) Let $\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$ be the sample mean. Derive $\hat{\theta}$, the method of moments estimator for θ , in terms of \bar{Y} . (3%)
2. The returns of an asset management firm in different years are independent and normally distributed with unknown mean and variance. The asset management firm claims that the standard deviation of the returns is as low as $\sigma = 2\%$ and the mean of the returns is $\mu = 22\%$. You believe that this is too good to be true. To verify your suspicion, you take the returns from the last 10 years. These are:

$$r = \{20.6, 19.2, 17, 19.1, 18.7, 22.5, 27.2, 17.9, 22.5, 21.3\}$$
 To help you with the calculations, we give:

$$\sum_{i=1}^{10} r(i) = 206 \text{ and } \sum_{i=1}^{10} (r(i) - \bar{r})^2 = 79.34$$
 Answer the following questions: (10%)
 - (1) Calculate the sample mean and standard deviation of the above random sample. (3%)
 - (2) What is the probability that when we draw a new random sample of size 10, its sample mean will be below the one you calculated in (1)? Assume that the company's claims are correct. (3%)
 - (3) What is the probability that when we draw a new random sample of size 10, its sample standard deviation will be larger than the one you calculated in (1), if that the company's claims are correct? (4%)

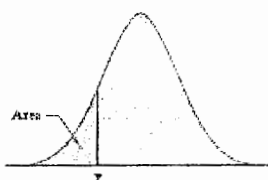
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3. (6%) Briefly describe the possible problems when using stepwise regression model as the “final” model.
4. (16%) Consider a multiple regression model for a response y , with one quantitative independent variable x_1 and one qualitative variable at three levels.
- (1) (5%) Provide a complete first-order model that includes the interaction between the quantitative and qualitative independent variables. You have to specify the coding scheme for dummy variables.
 - (2) (5%) Provide a complete second-order model that includes the interaction between the quantitative and qualitative independent variables.
 - (3) (3%) For (2), under what circumstances will the response curves of the model have the same shape but different y -intercepts?
 - (4) (3%) For (2), under what circumstances will the response curves of the model be identical?
5. (14%) To determine whether extra personnel are needed for the day, the owners of a water adventure park would like to find a model that would allow them to predict the day's attendance each morning before opening based on the day of the week and weather conditions. The model is of the form $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$, where $y =$ Daily admission,
- $$x_1 = \begin{cases} 1, & \text{if weekend} \\ 0, & \text{otherwise} \end{cases} \quad (\text{dummy variable})$$
- $$x_2 = \begin{cases} 1, & \text{if sunny} \\ 0, & \text{if overcast} \end{cases} \quad (\text{dummy variable}),$$
- $$x_3 = \text{predicted daily high temperature } (^\circ\text{F}).$$
- These data were recorded for a random sample of 30 days, and a regression model was fitted to the data. The least squares analysis produced the following results: $\hat{y} = -105 + 25x_1 + 100x_2 + 10x_3$ with $s_{\hat{\beta}_1} = 10$, $s_{\hat{\beta}_2} = 30$, $s_{\hat{\beta}_3} = 4$, $R^2 = .65$.
- (1) (5%) Is there sufficient evidence to conclude that this model is useful for the prediction of daily attendance? Use $\alpha = 0.1$.
 - (2) (5%) Is there sufficient evidence to conclude that the mean attendance increases on weekends? Use $\alpha = 0.10$.
 - (3) (4%) Use the model to predict the attendance on a sunny weekday with a predicted high temperature of 95°F .
6. (14%) Suppose a 3×3 factorial experiment is conducted with three replications. Assume that $SS(\text{total}) = 1000$. For the following scenario: The Sum of Squares of factor A main effect [$SS(A)$] is 20% of $SS(\text{total})$, the Sum of Squares for factor B main effect [$SS(B)$] is 10% of $SS(\text{total})$, and the Sum of Squares for interaction [$SS(AB)$] is 10% of $SS(\text{total})$.
- (1) (7%) Form an ANOVA table.
 - (2) (7%) Conduct the appropriate tests. Use $\alpha = 0.1$.

(背面仍有題目,請繼續作答)

Chi-Square (χ^2) Distribution
Area to the Right of Critical Value

Degrees of Freedom	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01
1	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892



Standard Normal Distribution

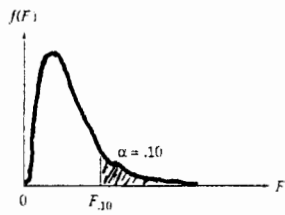
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0005	0.0008	0.0013	0.0020	0.0029	0.0040	0.0054	0.0072	0.0094
-3.3	0.0005	0.0007	0.0010	0.0016	0.0023	0.0032	0.0044	0.0059	0.0078	0.0101
-3.2	0.0007	0.0009	0.0013	0.0019	0.0027	0.0037	0.0049	0.0065	0.0084	0.0108
-3.1	0.0010	0.0013	0.0018	0.0025	0.0034	0.0045	0.0059	0.0076	0.0096	0.0119
-3.0	0.0013	0.0017	0.0023	0.0031	0.0041	0.0053	0.0068	0.0086	0.0106	0.0129
-2.9	0.0017	0.0022	0.0029	0.0038	0.0049	0.0062	0.0078	0.0097	0.0117	0.0140
-2.8	0.0022	0.0028	0.0036	0.0046	0.0058	0.0072	0.0089	0.0109	0.0129	0.0152
-2.7	0.0028	0.0035	0.0044	0.0055	0.0068	0.0083	0.0101	0.0121	0.0141	0.0164
-2.6	0.0035	0.0043	0.0053	0.0065	0.0079	0.0095	0.0114	0.0134	0.0154	0.0177
-2.5	0.0043	0.0052	0.0063	0.0076	0.0091	0.0108	0.0128	0.0148	0.0168	0.0191
-2.4	0.0052	0.0062	0.0074	0.0088	0.0104	0.0122	0.0143	0.0164	0.0184	0.0207
-2.3	0.0062	0.0073	0.0086	0.0101	0.0118	0.0137	0.0158	0.0179	0.0199	0.0222
-2.2	0.0073	0.0085	0.0100	0.0116	0.0134	0.0154	0.0175	0.0196	0.0216	0.0238
-2.1	0.0085	0.0100	0.0116	0.0133	0.0152	0.0173	0.0194	0.0215	0.0235	0.0257
-2.0	0.0100	0.0117	0.0134	0.0153	0.0173	0.0194	0.0215	0.0235	0.0255	0.0275
-1.9	0.0117	0.0135	0.0154	0.0174	0.0195	0.0216	0.0236	0.0256	0.0275	0.0294
-1.8	0.0135	0.0154	0.0174	0.0195	0.0216	0.0237	0.0257	0.0276	0.0295	0.0313
-1.7	0.0154	0.0174	0.0195	0.0216	0.0237	0.0258	0.0277	0.0296	0.0314	0.0332
-1.6	0.0174	0.0195	0.0216	0.0237	0.0258	0.0278	0.0297	0.0315	0.0333	0.0351
-1.5	0.0195	0.0216	0.0237	0.0258	0.0278	0.0298	0.0316	0.0334	0.0352	0.0369
-1.4	0.0216	0.0237	0.0258	0.0278	0.0298	0.0317	0.0335	0.0353	0.0370	0.0387
-1.3	0.0237	0.0258	0.0278	0.0298	0.0317	0.0336	0.0354	0.0371	0.0388	0.0404
-1.2	0.0258	0.0278	0.0298	0.0317	0.0336	0.0354	0.0372	0.0389	0.0405	0.0421
-1.1	0.0278	0.0298	0.0317	0.0336	0.0354	0.0372	0.0389	0.0406	0.0422	0.0437
-1.0	0.0298	0.0317	0.0336	0.0354	0.0372	0.0389	0.0406	0.0423	0.0438	0.0453
-0.9	0.0317	0.0336	0.0354	0.0372	0.0389	0.0406	0.0423	0.0439	0.0454	0.0469
-0.8	0.0336	0.0354	0.0372	0.0389	0.0406	0.0423	0.0439	0.0455	0.0470	0.0484
-0.7	0.0354	0.0372	0.0389	0.0406	0.0423	0.0439	0.0455	0.0471	0.0485	0.0499
-0.6	0.0372	0.0389	0.0406	0.0423	0.0439	0.0455	0.0471	0.0486	0.0500	0.0514
-0.5	0.0389	0.0406	0.0423	0.0439	0.0455	0.0471	0.0486	0.0501	0.0515	0.0528
-0.4	0.0406	0.0423	0.0439	0.0455	0.0471	0.0486	0.0501	0.0516	0.0529	0.0542
-0.3	0.0423	0.0439	0.0455	0.0471	0.0486	0.0501	0.0516	0.0530	0.0543	0.0556
-0.2	0.0439	0.0455	0.0471	0.0486	0.0501	0.0516	0.0530	0.0544	0.0557	0.0569
-0.1	0.0455	0.0471	0.0486	0.0501	0.0516	0.0530	0.0544	0.0558	0.0570	0.0582
0.0	0.0471	0.0486	0.0501	0.0516	0.0530	0.0544	0.0558	0.0571	0.0583	0.0595

系所組別： 資訊管理研究所甲組

考試科目： 統計學

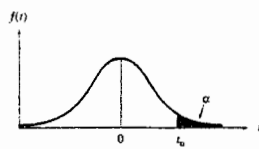
考試日期：0224，節次：3

※ 考生請注意：2 Table VII Percentage Points of the F-Distribution, $\alpha = .10$



ν_2	Numerator Degrees of Freedom								
	1	2	3	4	5	6	7	8	9
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63

Table V Critical Values of t



Degrees of Freedom	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	17.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.095	3.990	4.431
12	1.356	1.782	2.179	2.681	3.032	3.852	4.291
13	1.350	1.771	2.160	2.650	3.012	3.822	4.251
14	1.345	1.761	2.145	2.624	2.977	3.787	4.210
15	1.341	1.753	2.131	2.602	2.947	3.753	4.173
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.227	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.375
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291