

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please briefly answer the following questions (15%):
  - a. What is NP? (4%)
  - b. What is NP-hard? (4%)
  - c. What is NP-complete? (4%).
  - d. Name any NP-complete problem that you know of. (3%)
  
2.  $N$  jobs are to be scheduled for processing on one machine. Job  $i$ ,  $1 \leq i \leq N$  needs  $t_i$  units of processing time. If job  $i$  is finished by time  $T$ , where  $T$  is a given deadline, then a profit  $p_i$  is earned; otherwise, a penalty  $q_i$  is imposed. (Both  $p_i$  and  $q_i$  are positive integers.) We want to select a subset  $S$  of jobs such that  $\sum_{i \in S} t_i \leq T$  and  $f(S) = \sum_{i \in S} p_i - \sum_{i \notin S} q_i$ . (10%)
  - a. Show how to find such a set of jobs using dynamic programming. (5%)
  - b. Write pseudo code to solve your proposed dynamic program and analyze its time complexity. (5%)
  
3. Can a queue be implemented using stacks? Explain your answer. (5%)
  
4. Write a program to detect if a given singly linked list contains a cycle. Note, your program should avoid allocating extra memory space or using other data structures, such as a hash table. (10%)
  
5. Propose a data structure for representing sets and analyze the time complexity for the following operations: 1) adding a new element to a set; 2) taking the union of two sets; 3) finding the intersection of two sets; 4) finding the difference of two sets. (10%)

## 6. [28%] True or False, and EXPLAIN

Circle T or F for each of the following statements to indicate whether the statement is true or false, respectively. If the statement is correct, briefly state why. If the statement is wrong, explain why or give a counter example. Answers without reasons will get at most 1 point.

(a) [4%] ( T, F ) The solution of  $T(n) = T(n-1) + n^4$  is  $O(n^6)$

(b) [4%] ( T, F ) The algorithm for computing a Huffman code is an example of a greedy algorithm

(c) [4%] ( T, F ) Given an undirected connected simple graph  $G=(N,A)$ , where  $N$  and  $A$  represents the set of nodes and undirected arcs, respectively. Let  $c_{ij} \geq 0$  represent the length associated with each arc  $(i, j)$  in  $A$ . To detect whether  $G$  contains a zero-length cycle, it takes  $\Theta(m)$  time. You should explain how your method works so that it takes  $\Theta(m)$  time or not.

(d) [4%] ( T, F ) Based on the graph  $G=(N,A)$  in (c), suppose we construct a new graph  $G'=(N',A')$  isomorphic to the  $G$  in previous question (c), except setting new arc length  $d_{ij} = c_{ij}^2$  for each arc  $(i, j)$ . Any shortest path from  $u$  to  $v$  in  $G$  will still be a shortest path from  $u$  to  $v$  in  $G'$ .

(e) [4%] ( T, F ) Given a directed graph  $G$ , consider forming a graph  $G'$  as follows. Each vertex  $u' \in G'$  represents a strongly connected component (SCC) of  $G$ . There is an edge  $(u', v')$  in  $G'$  if there is an edge in  $G$  from the SCC corresponding to  $u'$  to the SCC corresponding to  $v'$ . Then  $G'$  is a directed acyclic graph.

(f) [4%] ( T, F ) A heap with  $n$  elements can be converted into a binary search tree in  $O(n)$  time.

(g) [4%] ( T, F ) A complete undirected bipartite graph  $K_{p,q}$  (with  $p>1$  and  $q>1$  nodes on one side) must contain an Euler path that passes through each edge exactly once with different origin and destination.

7. [22%] Possibly caused by the global warming, there are more disasters all over the world recently. In Taiwan there are typhoons or earthquakes which may damage the roads and make some villages in the mountain inaccessible from outside. In order to evacuate the people in the damaged area, we need to send out teams to repair the damaged roads. Suppose we draw a connected undirected network  $G=(N,A)$  that contains a set of  $|N|=n$  nodes representing  $n-1$  remote villages and 1 base, and a set of  $|A|=m$  undirected arcs denoting the roads connecting villages. Assume each arc  $(i, j)$  in  $A$  is broken and requires time  $t_{ij}$  to be repaired by at most one rescue team. If at time  $t$ , a path has immediately contained repaired arcs (i.e., no more broken) from the base to a node (i.e., village), we say those people on that node are rescued at time  $t$ . Answer the following questions: (please see the next page)

- (a) [10%] If there is only one rescue team available in the base node, and the objective is to rescue all the people in villages so that the waiting time to be rescued for the last village can be minimized. How to do it? Give a method, briefly explain its procedures and its complexity.
- (b) [12%] Assuming you have  $n$  rescue teams of the same capability and efficiency to repair roads, answer the same questions in (a) (i.e., how to assign your teams to rescue all the people so that the last people can be rescued in minimum waiting time)