國立成功大學

112學年度碩士班招生考試試題

編　號：239

系　所：資訊管理研究所

科　目：資料結構

日　期：0207

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Part A.

I. Choose the alternative that best completes the statement or answers the question. 12%; each 3%

1. How many linked lists are used to represent a graph with 8 nodes and 10 edges, when using an edge list representation?
(a) 8
(b) 10
(c) 18
(d) 80
(e) none of the above

2. What is the range of values computed by the hash function Hash(X) = X mod 1000?
(a) 0 to 1000
(b) 1 to 1000
(c) 0 to 999
(d) 1 to 999
(e) none of the above

3. Which of the following sorting algorithm has the same average and worst-case time bounds (big-*O* notation) as heapsort?
(a) insertion sort
(b) mergesort
(c) quicksort
(d) shellsort
(e) none of the above

4. Which of the following does the binary heap implement?
(a) binary search tree
(b) priority queue
(c) hash table
(d) queue
(e) stack

II. Short answer questions and discussion

5. Given a tree, T, its height is defined as the length of the longest root-to-leaf path in T.
(1) [5%] What is the maximum number of nodes in a binary tree of height 8?
(2) [5%] What is the minimum number of nodes in a binary tree of height 9?

6. [5%] The following numbers are inserted sequentially into a binary search tree: 5 2 4 3 8 1 7 9. What is the number in the deepest node?

7. Tom is planning a trip in Europe. He plans to visit n countries. Assuming there exist exactly K multiple direct flights between each country pair i & j, with different costs and flight time. Answer the following questions.
(1) [4%] If we use a node to represent a country and a directed arc to represent a direct flight from one country to another (but not to itself). How many arcs will this graph contain?
(2) [7%] Based on (1), please explain how (by the Adjacency matrix or the Adjacency list) you may store this graph. Also, explain why you choose one but not the other of the 2 data structures.
(3) [7%] If Tom plans to visit all these n countries at minimum costs. Can this be done by a 1-ALL shortest path problem or by a Minimum-spanning tree? Please explain your answer.
(4) [5%] If Tom plans to visit L other countries starting from France (one of the n countries) and then return to France with total costs no more than Q and total flight time no more than P. Can this be done in polynomial time if $L < 7$? Explain how and why.

III、8、[20%] **True** or **False**, and **EXPLAIN**

Circle **T(true)** or **F(false)**. If the statement is correct, ***briefly state why***. If the statement is wrong, ***explain why or give a counterexample***. Answers **WITHOUT** reasons will get **at most 1 point**.

(a) [4%] If we have stored n random real numbers in a 1-D array. If these numbers are known to be uniformly distributed in [0, 1000]. To sort these n numbers in increasing order requires $\Omega(n \log n)$ time.

(b) [4%] If we use a binary search tree (BST) to store $n \geq 3$ distinct integers, printing these n integers out and storing them by a min-heap takes $\omega(n \log n)$ time.

(c) [4%] Given a directed acyclic graph containing negative arc weight, if we multiply each arc weight with "-1", we can use Dijkstra's algorithm to calculate a longest directed path from node s to node t.

Let G be a simple digraph (no self-loops or parallel edges) of n nodes and m edges with positive and distinct edge weights.

(d) [4%] If we want to answer the question of whether G contains a directed cycle that must pass arc (s,t) with length at most Q (a given constant), this cannot be done in polynomial time because there may exist exponentially many such kinds of cycles.

(e) [4%] Suppose Tom stores n unsorted distinct real numbers by a Red-Black Tree, and Mary stores them by an 1-D array. Now Tom and Mary want to print out the median. Both of them would take $O(n)$ time.

IV. 9、[30%] John is responsible for designing the layout of his factory, which is a $U \times V$ grid network containing UV squares (each square has a unit side length). In particular, the factory is a rectangle with horizontal length V units and vertical length U units. Now John wants to install n machines inside this factory, where machine i is a rectangle of v_i units horizontally and u_i units vertically (assuming each machine will not be rotated). Let the coordinates of the left-up, right-up, left-down, and right-down corners of this factory to be (0,0), (0,V), (U,0), and (U,V). Suppose the coordinate of the center for machine i is (x_i, y_i), and we need to keep a **safety distance of *d* units** both horizontally and vertically between the "**boundary**" of any two machines.

(a) [20%] Suppose John already has (x_i, y_i) for each machine i. Give a method to help John determine whether the given coordinates satisfy constraints:(i) all machines must be inside the factory, and (ii) all machines must keep a safety distance between each other. For each of the above (i)(ii) checks, you should explain how your methods work and their complexity.

(b) [10%] Suppose John has a feasible center coordinate for each machine i, and installing a wire to connect machine i and j costs $c_{ij} = c_{ji}$. He wants to install wires between machines so that all machines are **connected with minimum total costs**. (i) Can this be done in polynomial time? If yes, explain how and why; otherwise, explain why not. (ii) answer the same questions in (i) if the final graph must be a **cycle**.