

1. (6%) For following two figures of two populations, one is skew to the left and the other is skew to the right. Please draw mean, median, and mode on each figure.



2. (6%) An automobile insurance company estimates the following loss probabilities for the next year on a \$1,000,000 sports car:

Total loss: 0.001

50% loss: 0.01

25% loss: 0.05

10% loss: 0.10

Assuming the owner needs to pay the first \$25,000 damage, how much the annual premium should the company charge in order to average \$1,500 profit per policy sold?

3. The Environmental Protection Agency (EPA) estimated that the 2000 T1 car has a mean of 35 miles per gallon (mpg) on the highway. However, the T1 car company claims that the EPA has underestimated the T1 car's mpg. Thus the T1 Car Company has done the similar test-driving with randomly selected 36 cars and gets average 37.3 mpg, standard error of 6.4 mpg.

a. (6%) If the T1 car company wishes to show that the mean mpg is greater than 35 mpg, what should the null and alternative hypotheses?

b. (6%) Do the data provide sufficient evidence to support the T1 car company's claim? Please test using $\alpha=0.05$ and list any assumptions you make in conducting the test.

c. (8%) Please calculate the power of the test for the mean value of 35.5, 36.0, 36.5, 37.0, 37.5, 38.0, assuming $s=6.4$ mpg is a good estimate of σ .

d. (5%) Please draw the power of the test curve and use this curve to approximate the power of test when $\mu=40$. What conclusion can you get as far as power of test and hypothesis testing concern if the true value of the mean mpg for this car is really 40?

4. Suppose a random variable x has the following probability distribution function:

$$f(x) = \frac{1}{\sqrt{0.5\pi}} e^{-2x^2+8x-8}, \quad -\infty < x < \infty$$

a. (3%) Please find $P(-5 \leq x \leq -2)$.

b. (3%) Please find $P(|x+3| \leq 1)$.

5. (7%) Suppose you want to estimate the difference between population proportions correct to within 3% with confidence coefficient 0.9. You think both p_1 and p_2 are near 0.4, and you want to select samples of equal size from the two populations. Please find the required sample sizes, n_1 and n_2 .

(背面仍有題目,請繼續作答)

6. (12%) A straight line model $y_i = \beta_1 x_i + \varepsilon_i$ was fit to the following data,

x	1	2	2	3	3	3	4	6	6	7	8	10	10
y	6	64	44	104	94	134	144	264	284	294	314	334	314

The fitted model is $\hat{y} = 37x$ with $\hat{\sigma}^2 = 1322$.

- 1). Show that the regression model is significant. (3%)
 - 2). Calculate the residuals. (3%)
 - 3). Plot the residuals to investigate the adequacy of the fitted model. What is the inadequacy? (3%)
 - 4). Confirm the inadequacy (or adequacy) with a lack of fit test (ANOVA). (3%)
7. (12%) A time series process is considered to be of the form $X_t = b + \varepsilon_t$, where b is a constant and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. The process has been observed for T periods and X_1, \dots, X_T have been recorded.
- 1). Develop the least square estimator for b . (3%)
 - 2). Estimate σ_ε^2 . (3%)
 - 3). What is the forecasting equation for X_{T+r} ? (3%)
 - 4). Develop a $100(1-\alpha)\%$ prediction interval for X_{T+r} . (3%)

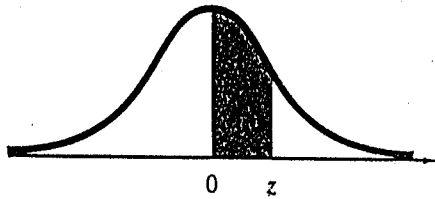
8. (13%) 1. A series of experiments are designed and performed to determine the difference of two production processes A and B to manufacture the 1.8 liter four-cylinder engine head. There are two batches of aluminum-alloy material for the experiment. Each batch includes ten samples. The results of one important response are summarized as follows:

Process A	7	3	3	4	8	3	2	9	5	4
Process B	6	3	5	3	8	2	4	9	4	5

All statistical tests are to be performed at the significant level $\alpha = 5\%$.

- 1). An independent t-test is designed for this experiment. Write down the assumptions, hypothesis, perform appropriate analysis and draw conclusion. (4%)
 - 2). It was feared that the batch to batch material variation in the production processes would influence the analysis, propose an alternative (the simplest) test for this experiment. Write down the assumptions, hypothesis, perform appropriate analysis and draw conclusion. (4%)
 - 3). Based on 1) and 2), construct confidence intervals for the difference of response from two production processes and draw conclusion. (5%)
9. (13%) For the last problem, the investigator choose the analysis of variance with one single factor as the experimental design (with two levels and 10 observations each). All statistical tests are to be performed at the significant level $\alpha = 5\%$.
- 1). Write down the statistical model, assumptions, hypothesis, perform appropriate analysis, and draw conclusion. (4%)
 - 2). It was feared that the batch to batch material variation in the production processes would influence the analysis, a completely randomized block design is selected in this experiment, with the batch as blocking factor. Write down the assumptions, hypothesis, perform appropriate analysis and draw conclusion. (4%)
 - 3). Which test is the best strategy for this experiment? Explain! Have you reached the same conclusion for the tests performed in 1) and 2)? Why? (5%)

Normal Curve Areas



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Source: Abridged from Table I of A. Hald, *Statistical Tables and Formulas* (New York: Wiley), 1952. Reproduced by permission of A. Hald.

(背面仍有題目,請繼續作答)

