

1. (10%) A manager is planning to buy a new machine of either type A or type B . For each day's operation the number of repairs X that machine A requires is a Poisson random variable with mean $0.10t$, where t denotes the time (in hours) of daily operation. The number of daily repairs Y for machine B is Poisson with mean $0.12t$. The daily cost of operating A is $C_A(t) = 10t + 30X^2$; for B it is $C_B(t) = 8t + 30Y^2$. Assume that the repairs take negligible time and each night the machines are to be cleaned, so they operate like new machines at the start of each day. Which machine minimizes the expected daily cost if a day consists of (1) ten hours? (2) twenty hours?
2. (10%) Let two random variables, X and Y , have joint density

$$f(x_1, x_2) = \begin{cases} 1/2 & 0 \leq x_1 \leq x_2, 0 \leq x_2 \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the conditional expectation of X_1 , given that $X_2 = 1$.

3. If X is a random variable with moment-generating function $M(t)$, and Y is a function of X given by $Y = aX + b$, (1) (5%) show that the moment-generating function for Y is $e^{bt}M(at)$. (2) (5%) Use the result to show that $E(Y) = aE(X) + b$.
4. (1) (3%) Describe the difference between deterministic model and probabilistic model in mathematical forms.
 (2) (3%) For a simple regression model, describe how to statistically check the adequacy of the model.
 (3) (4%) For a simple regression line, describe how to measure the contribution of x in predicting Y in mathematical ways.
5. (1) (6%) A student wants to fit the following second-order model

$$E(Y) = \beta_0 + \beta_1x + \beta_2x^2$$

A set of data (x, y) are collected as $(10, 15)$, $(10, 16)$, $(10, 17)$, $(20, 31)$, $(20, 32)$, and $(20, 35)$. What problems does this student have in fitting the model?

- (2) (4%) An economist claims that he has established a regression model to predict the next year's growth in GNP based on rate of inflation, consumer price index, and other factors using the data last year. Please indicate the possible problems in this matter.

6. Please explain the following terms in detail: (20%)
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|------------------------|--|
| a. Sample Statistic | b. Sample Estimator |
| c. Confidence Interval | d. Degree of Freedom |
| e. Power Curve | f. Residual Analysis |
| g. Outlier | i. Moving Averages vs. Exponential Smoothing |
| j. States of Nature | k. Unbiasedness |
7. Please State the differences between the t -test and the ANOVA, i.e., under what kinds of situations, and for what objectives, will we use these methods? Also, state the testing processes. If we have the concern of blocking effects, then what adjusted designs for these two methods should be performed? and How? (15%)
8. Why do we need a non-parametric analysis? Please state the pros and cons. One of the famous non-parametric analyses is called Mann-Whitney-Wilcoxon Test. Please state the assumptions and the testing procedure for it. (You may give an example for demonstration) (15%)