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1. (13%) An random variable X has the following p.d.f.：

$$f(x)=\begin{cases}2x^{-3}, & \text{if } x\geq 1\\ 0, & \text{otherwise.}\end{cases}$$

a. (3%) Find the c.d.f. of X.

b. (5%) Give a formula for the *p* th quantile of X and use it to find the median of X.

c. (5%) Find the mean and variance of X.

2. (10%) Let $\overline{X}$ be the mean of a random sample of size n from an $N(\mu,\sigma^2)$ distribution and suppose that σ^2 is known.

a. (6%) Show that

$$\left[\overline{X}-z_{\alpha_1}\frac{\sigma}{\sqrt{n}},\overline{X}+z_{\alpha_2}\frac{\sigma}{\sqrt{n}}\right]$$

is a $(1-\alpha)$ confidence interval for μ if α_1 and α_2 satisfy $\alpha_1+\alpha_2=\alpha$.

b. (4%) What does the above confidence interval look like when $\alpha_1=\alpha,\ \alpha_2=0$? This is so called lower one-sided confidence bound for μ.

3. (27%) You are an industrial engineer at a potato chip company. Your market research suggests that when bags of chips appear to be less than 1/2 full, customers complain. It has thus been decided that for a one foot high bag, the average height, μ, of the chips in the bag must exceed 6.5 inches and the standard deviation, σ, must be about 1/2 inch.

A line supervisor has collected data throughout the month of January by taking 4 bags from the line each Monday, Wednesday, and Friday and measuring the height of the chips in the bag. He wants to run a hypothesis test and wants you to interpret it for him.

One-Sample T: C50

Variable	N	Mean	StDev	SE Mean
C50	48	6.63716	0.47834	0.06904

More chips must be put into the bag *unless* the data reject the idea that μ is equal or less than 6.5 at a 95% confidence level. For parts a)-e) assume that the data are independent and identically distributed with a normal distribution. Also it is given that $x^2_{0.025,47}=67.82$ and $x^2_{0.975,47}=29.95$.

a. (2%) State the null and alternative hypotheses.

(背面仍有題目,請繼續作答)

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b. (5%) Based on this information, do more chips need to be put in the bag? Justify your answer.

c. (5%) Make a two-sided 95% confidence interval for σ. Do you have any good reason to doubt that the standard deviation is close to 1/2 inch?

d. (5%) You decide that you will assume that the actual standard deviation of the line is 1/2 inch. Each week, you will randomly draw 48 samples off the line throughout the week and run a hypothesis test. Unless the test rejects the idea that μ is equal or less than 6.5 at a 95% confidence level (now assuming that the actual standard deviation is 1/2), then you will add more chips the next week. The line supervisor explains that there is concern that this test may require the plant to put in more chips when the average height is already above 6.5 inches. Is the supervisor worried about Type I or Type II error? Explain.

e. (5%) Suppose that you run the test that you set up in d). How high does the sample mean (the average of the 48 bags that you chose) have to be to keep you from putting in more chips?

f. (5%) If you implement the procedure that you set up in e) and the actual process mean for the week is 6.7 inches, what is the probability that you will have to add more chips the following week?

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4. (6%) State the assumptions for analysis of variance.

5. (10%) Interpret the multiple coefficient of determination R^2 and adjusted multiple coefficient of determination R^2_{adj}. State the difference between them when adding the independent variables in a multiple regression model.

6. (10%) Five different auditing procedures were compared in terms of total audit time. To control for possible variation due to the person conducting the audit, four accountants were selected randomly and treated as blocks in the experiment. The following values were obtained by the ANOVA procedure: the total sum of squares (SST) = 100, the sum of squares due to treatments (SSTR) = 45, the sum of squares due to blocks (SSBL) = 36. Use α= .05 to test

a. whether there is significant difference in the mean total audit time for the five auditing procedures.
b. whether there exists the consistency of the four accountants.

Note: $F_{0.05}(5,20) = 2.71$, $F_{0.05}(5,12) = 3.11$, $F_{0.05}(4,12) = 3.26$, $F_{0.05}(4,3) = 9.12$, $F_{0.05}(3,4) = 6.59$, $F_{0.05}(3,12) = 3.49$, $F_{0.05}(3,20) = 3.1$

7. (12%) Consider a regression study involving a dependent variable y, a quantitative independent variable x_1, and a qualitative variable with two levels (level 1 and level 2).

a. Write a multiple regression equation relating x_1 and the qualitative variable to y.
b. What is the expected value of y corresponding to level 1 of the qualitative variable?
c. What is the expected value of y corresponding to level 2 of the qualitative variable?
d. Interpret the parameters in your regression equation.

8. (12%) In a regression analysis involving 30 observations, the following estimated regression equation was obtained.

$$\hat{y} = b_0 + b_1x_1 - b_2x_2 + b_3x_3 + b_4x_4$$

For this estimated regression equation SST = 1805 and SSR = 1760.

a. At α= .05, test the significance of the relationship among the variables.

Suppose variables x_1 and x_4 are dropped from the model and the following estimated regression equation is obtained.

$$\hat{y}' = b_0' - b_2'x_2 + b_3'x_3$$

For this model SSR = 1705.

b. Compute SSE(x_1, x_2, x_3, x_4).
c. Compute SSE(x_2, x_3).
d. Use an F test and a .05 level of significance to determine whether x_1 and x_4 contribute significantly to the model.

Note: $F_{0.05}(4,29) = 2.70$, $F_{0.05}(4,25) = 2.76$, $F_{0.05}(2,29) = 3.33$, $F_{0.05}(2,25) = 3.39$

系所組別： 資訊管理研究所甲組

考試科目： 統計學

考試日期：0306，節次：3

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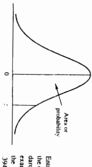
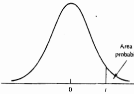
TABLE 1 STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean and z is .3944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

TABLE 2 t DISTRIBUTION



Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail .20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
10	.879	1.372	1.812	2.228	2.764	3.169
11	.876	1.363	1.796	2.201	2.718	3.106
12	.873	1.356	1.782	2.179	2.681	3.055
13	.870	1.350	1.771	2.160	2.650	3.012
14	.868	1.345	1.761	2.145	2.624	2.977
15	.866	1.341	1.753	2.131	2.602	2.947
16	.865	1.337	1.746	2.120	2.583	2.921
17	.863	1.333	1.740	2.110	2.567	2.898
18	.862	1.330	1.734	2.101	2.552	2.878
19	.861	1.328	1.729	2.093	2.539	2.861
20	.860	1.325	1.725	2.086	2.528	2.845
21	.859	1.323	1.721	2.080	2.518	2.831
22	.858	1.321	1.717	2.074	2.508	2.819
23	.858	1.319	1.714	2.069	2.500	2.807
24	.857	1.318	1.711	2.064	2.492	2.797
25	.856	1.316	1.708	2.060	2.485	2.787
26	.856	1.315	1.706	2.056	2.479	2.779
27	.855	1.314	1.703	2.052	2.473	2.771
28	.855	1.313	1.701	2.048	2.467	2.763
29	.854	1.311	1.699	2.045	2.462	2.756

TABLE 2 t DISTRIBUTION (*Continued*)

Degrees of Freedom	Area in Upper Tail .20	.10	.05	.025	.01	.005
35	.852	1.306	1.690	2.030	2.438	2.724
36	.852	1.306	1.688	2.028	2.434	2.719
37	.851	1.305	1.687	2.026	2.431	2.715
38	.851	1.304	1.686	2.024	2.429	2.712
39	.851	1.304	1.685	2.023	2.426	2.708
40	.851	1.303	1.684	2.021	2.423	2.704
41	.850	1.303	1.683	2.020	2.421	2.701
42	.850	1.302	1.682	2.018	2.418	2.698
43	.850	1.302	1.681	2.017	2.416	2.695
44	.850	1.301	1.680	2.015	2.414	2.692
45	.850	1.301	1.679	2.014	2.412	2.690
46	.850	1.300	1.679	2.013	2.410	2.687
47	.849	1.300	1.678	2.012	2.408	2.685
48	.849	1.299	1.677	2.011	2.407	2.682
49	.849	1.299	1.677	2.010	2.405	2.680
50	.849	1.299	1.676	2.009	2.403	2.678
51	.849	1.298	1.675	2.008	2.402	2.676
52	.849	1.298	1.675	2.007	2.400	2.674
53	.848	1.298	1.674	2.006	2.399	2.672
54	.848	1.297	1.674	2.005	2.397	2.670
55	.848	1.297	1.673	2.004	2.396	2.668
56	.848	1.297	1.673	2.003	2.395	2.667
57	.848	1.297	1.672	2.002	2.394	2.665
58	.848	1.296	1.672	2.002	2.392	2.663
59	.848	1.296	1.671	2.001	2.391	2.662
60	.848	1.296	1.671	2.000	2.390	2.660
61	.848	1.296	1.670	2.000	2.389	2.659
62	.847	1.295	1.670	1.999	2.388	2.657
63	.847	1.295	1.669	1.998	2.387	2.656
64	.847	1.295	1.669	1.998	2.386	2.655
65	.847	1.295	1.669	1.997	2.385	2.654
66	.847	1.295	1.668	1.997	2.384	2.652
67	.847	1.294	1.668	1.996	2.383	2.651
68	.847	1.294	1.668	1.995	2.382	2.650
69	.847	1.294	1.667	1.995	2.382	2.649
70	.847	1.294	1.667	1.994	2.381	2.648
71	.847	1.294	1.667	1.994	2.380	2.647
72	.847	1.293	1.666	1.993	2.379	2.646
73	.847	1.293	1.666	1.993	2.379	2.645
74	.847	1.293	1.666	1.993	2.378	2.644