系所組別： 財務金融研究所

考試科目： 微積分

※ 考生請注意：本試題 ☐可 ☑不可 使用計算機

一、選擇題 50 分(每題五分)

1. $\lim\limits_{x\to\infty}\frac{1}{\sqrt{x}}\left[\frac{1}{\sqrt{x}+1}+\frac{1}{\sqrt{x}+2}+\cdots+\frac{1}{\sqrt{x}+\sqrt{x}}\right]=$ (a) 0 (b) $2\ln 2-2$ (c) $\ln 2-2$ (d) $2-2\ln 2$

2. $\lim\limits_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+1}=$ (a) 0 (b) ∞ (c) $\exp(8)$ (d) $\exp(-8)$

3. $f(x)=(x-1)^7e^x$ 求 $f^{(20)}(1)=$ (a) $\frac{e(20!)}{13!}$ (b) $\frac{e(21!)}{17!}$ (c) $\frac{e(21!)}{13!}$ (d) $\frac{e(20!)}{17!}$

4. $\lim\limits_{x\to 0^+}\frac{\frac{\pi}{2}-\tan^{-1}\frac{1}{x}}{x}=$ (a) 0 (b) ∞ (c) $-\infty$ (d) 1

5. $\lim\limits_{x\to 0^-}\frac{\frac{\pi}{2}-\tan^{-1}\frac{1}{x}}{x}=$ (a) 0 (b) ∞ (c) $-\infty$ (d) 1

6. $\lim\limits_{n\to\infty}\frac{x-\sin 2x}{\tan x}=$ (a) 0 (b) 1 (c) -1 (d) ∞

7. $\int\frac{2x^2+x-4}{x^3-x^2-2x}dx=$
   (a) $2\ln|x|+\frac{1}{3}\ln|x-2|-\frac{1}{3}\ln|x+1|+c$
   (b) $2\ln|x|-\frac{1}{3}\ln|x-2|+\frac{1}{3}\ln|x+1|+c$
   (c) $2\ln|x|+\frac{1}{3}\ln|x+2|-\frac{1}{3}\ln|x-1|+c$
   (d) $2\ln|x|-\frac{1}{3}\ln|x-2|-\frac{1}{3}\ln|2x+1|+c$

8. $\lim\limits_{n\to\infty}\frac{e^{\frac{1}{n}}+e^{\frac{2}{n}}+\cdots+e^{\frac{n}{n}}}{n}=$ (a) e (b) e^2 (c) $\frac{e}{2}-1$ (d) $e-1$

9. Find constants a and b such that the function: $f(x)=\begin{cases}12, & x\le -3\\ ax+b, & -3<x<5\\ -12, & x\ge 5\end{cases}$ is continuous on the entire real line. (a) a = 3, b = 0 (b) $a=-3,\ b=3$ (c) $a=3;\ b=-3$ (d) $a=-3,\ b=-3$

(背面仍有題目,請繼續作答)

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10. Evaluate $\int \tan^6 x dx =$ (a) $\frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \tan x - x + c$

(b) $\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + \tan x + x + c$

(c) $\frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x - \tan x - x + c$

(d) $-\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + \tan x - x^2 + c$

二、非選擇題 50 分

1. (10%) Show that $\int_0^\infty e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$

2. (10%) Evaluate $\frac{d}{dx}\left(e^x \ln\left(\sin^{-1} x^2\right)\right)$

3. (30%) The assumptions for the Black–Scholes–Merton option valuation are as follows:

(1) The stock price follows the geometric Brownian motion (GBM).
(2) The short selling of securities with full use of proceeds is permitted.
(3) There are no transactions costs or taxes. All securities are perfectly divisible.
(4) There are no dividends during the life of the derivative.
(5) There are no riskless arbitrage opportunities.
(6) Security trading is continuous.
(7) The risk-free rate of interest, r, is constant and the same for all maturities.

The formula for the price c_t of a European call option at time 0 on a non-dividend-paying stock is given as

$$c = SN(d_1) - Ke^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = d_2 + \sigma\sqrt{T}$$

and K, r, T, and σ are the strike price, interest rate, time to maturity and volatility, respectively.

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Questions:

(a) (10%) Show that the delta ($\frac{\partial c_t}{\partial S_t}$) of the call price is $N(d_1)$.

(b) (10%) Show that the theta ($\frac{\partial c_t}{\partial T}$) of the call price is

$$-\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT} N(d_2).$$

(c) (10%) Show that the vega ($\frac{\partial c_t}{\partial \sigma}$) of the call price is $S_0 \sigma \sqrt{T} N'(d_1)$

Hint:

$$\frac{d}{dx}\left[\int_{b(x)}^{a(x)} f(x,t)\,dt\right] = f\left(x,a(x)\right)\frac{d}{dx}\left[a(x)\right] - f\left(x,b(x)\right)\frac{d}{dx}\left[b(x)\right] + \int_{b(x)}^{a(x)}\left[\frac{df(x,t)}{dx}\right]dt$$