請勿在本試題紙上作答，否則不予計分

※ 考生請注意：本試題可使用計算機 (考生必需詳列其推論過程，否則不與計分)

1. Let $x_1, x_2, ..., x_n$ be a sample of size n from a normal distribution $N(u, \sigma^2)$. Consider the following point estimator of u:

$\hat{u}_1 = \bar{x}$, the sample mean

$\hat{u}_2 = x_1$

$\hat{u}_3 = \frac{x_1}{2} + \frac{1}{2(n-1)}(x_2 + x_3 + ... + x_n)$

(a) Which of these are unbiased? (3%)
(b) Which of these are consistent? (2%)
(c) Find the relative efficiencies of the three estimators. (5%)
(d) Are all unbiased estimators consistent? Why? (5%)
(e) Make a comparative analysis among the three desirable properties of estimators. (5%)

2. In Question 1 consider the point estimator of σ^2:

$$\hat{\sigma}_1^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\hat{\sigma}_2^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\hat{\sigma}_3^2 = (x_1 - \bar{x})^2$$

Which of these estimators are
(a) Unbiased? (5%)
(b) Consistent? (5%)

(背面仍有題目，請繼續作答)

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3. An electronics firm has three plants, one in Taiwan, one in Singapore and one in the United States. The firm's vice president of manufacturing is interested in whether or not output per hour of labor differs among the plants. Fifteen workers are chosen at random from each plant, and the output per hour of each worker is measured. The mean and sample stander deviation of these workers' output per hour of labor are shown below:

	Singapore	Taiwan	United States
Mean	15.1	16.2	13.4
Standard Deviation	3.2	4.0	3.1

(a) In which plant does output per hour of labor seem highest? In which one does it seem lowest? (2%)
(b) Construct the relevant analysis-of-variance (ANOVA) table. (5%)
(c) Can the observed differences among the plants in the mean output per hour of labor be attributed to chance? (Let α=0.05, and $F_{0.05}$=3.23). (3%)
(d) Construct 95% confidence intervals for the differences among the means. (5%)
(e) What assumptions and limitations are involved in your answer to (c)? (10%)

4. Consider two separate variables, x and y.
(a) Show the formula for the sample variance of x and y. (2%)
(b) Show the formula for the sample covariance and correlation between x and y. (3%)
(c) Make a comparative analysis among variance, covariance and correlation. (5%)
(d) If one student uses the aforementioned formulas to calculate the variances and correlations of the risk factors (e.g., stock price, interest rate and foreign exchange rate) over the last few months and assume that tomorrow's changes in risk factors will come from a distribution that has the same variance and correlation as experienced historically. Is his assumption reasonable? Why? What are the solutions? (10%)

5. Consider the linear regression model:

$$y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots\ldots + \beta_k x_{ki} + u_i \quad i = 1, 2, \ldots, n$$

(a) Explain how to obtain the estimators using the least squares method. (5%)
(b) Clearly list the assumptions involved in the traditional regression model. (5%)
(c) Are these assumptions are reasonable in practice? For each unreasonable assumption, provide an illustrative example. (5%)
(d) Address a comparative analysis between Regression and ANOVA. (10%)