國立成功大學 105 學年度碩士班招生考試試題

系　　所：財務金融研究所

考試科目：統計學　　　　考試日期：0228，節次：3

※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Multiple choice (30%)

1. If (P(X=x|Y=y) = P(X=x), then

a.) Y is the dependent variable

b.) X and Y are positively correlated

c.) X and Y are statistically independent

d.) Y must be a discrete random variable

2. The expected value of a random variable is

a.) the probability weighted mean

b.) a measure of central tendency of the pdf

c.) average value that occurs in many repeated trial of an experiment

d.) all of the above

3. If Z is a random variable generated by adding together X and Y which are also random variables, what do we know about var(Z) if X and Y are positively correlated.?

a.) var(Z) = var(X) + var(Y)

b.) var(Z) < var(X) + var(Y)

c.) var(Z) > var(X) + var(Y)

d.) var(Z) = var(X) * var(Y)

4. Which of the following is NOT an assumption of the Simple Linear Regression Model?

a.) The value of y, for each value of x, is

$$y = b_1 + b_2 x + e$$

b.)The variance of the random error e is

$$var(e) = \sigma^2$$

c.) The covariance between any pair of random errors e_i and e_j is zero

d.) The parameter estimate of b_1 is unbiased.

5. In the OLS model, what happens to var(b_1) as the sample size (N) increases?

a.) it also increases

b.) it decreases

c.) it does not change

d.) cannot be determined without more information

6. Which of the following non-linear adjustments CANNOT be accommodated using OLS?

a.) including an independent variable that has been raised to a power

b.) taking a logarithmic transformation of the dependent variable

c.) including a binary indicator variable
d.) raising parameters to a power

7. How do you interpret the estimated value of b_2 in the following equation:
$\ln(ENT\_EXP) = b_1 + b_2 (INCOME) + e$
where *INCOME* is annual household income (in thousands) and *ENT_EXP* is annual entertainment expenses?
a.) the income elasticity of entertainment
b.) when multiplied by 100 it is the percentage increase in entertainment expenses associated with an additional $1000 in income
c.) the increase in entertain expenses associated with a 1% increase in income
d.) the average of the logarithm of entertainment expenses for a household with zero income

8. You have estimated the following equation using OLS:
$\hat{y} = 33.75 + 1.45\, MALE$
where y is annual income in thousands and *MALE* is an indicator variable such that it is 1 for males and 0 for females. According to this model, what is the average income for females?
a.) $33,750
b.) $35,200
c.) $32,300
d.) cannot be determined

9. For which alternative hypothesis do you reject H_0 if $t \le t_{(\alpha, N-2)}$?
a.) $\beta_k = c$
b.) $\beta_k \neq c$
c.) $\beta_k > c$
d.) $\beta_k < c$

10. When should a left-tailed significance test be used?
a.) When economic theory suggests the coefficient should be positive
b.) When it allows you to reject the null hypothesis at a lower p-value
c.) When economic theory suggests the coefficient should be negative
d.) When you know the true value of β_k is positive.

Problems:

1.

Consider the following five observations for regression analysis. (30%)

x	y	$x-\bar{x}$	$(x-\bar{x})^2$	$y-\bar{y}$	$(x-\bar{x})(y-\bar{y})$
0	6				
1	5				
2	3		(請勿在此作答)		
3	1				
4	0				
$\sum x_i$	$\sum y_i$	$\sum(x_i-\bar{x})$	$\sum(x_i-\bar{x})^2$	$\sum(y_i-\bar{y})$	$\sum(x-\bar{x})(y-\bar{y})$

a. Complete the entries in the table. Put the sums in the last row. What are the sample means $\bar{x}$ and $\bar{y}$? (3%)

b. If the regression model is: $y = b_1 + b_2x + e$，using the results of the least square method to calculate b_1 and b_2. (3%)

c. Using the numerical values, show that
$\sum(x_i-\bar{x})^2 = \sum x_i^2 - N\bar{x}^2$ and $\sum(x-\bar{x})(y-\bar{y}) = \sum x_iy_i - N\bar{x}\bar{y}$. (3%)

d. Use the least squares estimates from b., to complete the following entries. Put the sums in the last row. (3%)

x_i	y_i	$\hat{y}_i$	$\hat{e}_i$	$\hat{e}_i^2$	$x_i\hat{e}_i$
0	6				
1	5				
2	3	(請勿在此作答)			
3	1				
4	0				
		$\sum\hat{y}_i$	$\sum\hat{e}_i$	$\sum\hat{e}_i^2$	$\sum x_i\hat{e}_i$

e. Plot the data point and sketch the fitted regression line. (3%)

f. On the sketch in part e, locate the point of means $(\bar{x}, \bar{y})$，does your fitted line pass through that point? (3%)

g. Show that for these numerical values $\bar{y} = b_1 + b_2\bar{x}$. (3%)

h. Show that for these numerical values $\bar{\hat{y}} = \bar{y}$. (3%)

i. Compute $\hat{\sigma}^2$ (variance of $\hat{e}_i$)? (3%)

j. Compute variance of $\widehat{b_2}$. (3%)

2. (20%)

Let x be a continuous random variable with probability density function given by

$$f(x) = -\frac{1}{2}x + 1,\ \ 0 \le x \le 2.$$

a. Graph the density function f(x). (3%)

b. Find the total area beneath f(x) for $0 \le x \le 2$. (3%)

c. Find $P(X \ge 1)$ using both geometry and integration. (3%)

d. Find $P(X \le \frac{1}{2})$ using both geometry and integration. (3%)

e. Find $P(X = 1\frac{1}{2})$. (2%)

f. Find the expected value and variance of X. (3%)

g. Find the cumulative distribution function of X. (3%)

3. (20%)

Suppose that Y_1, Y_2, Y_3 is a random sample from a $N(\mu, \sigma^2)$ population. To estimate μ, consider the weighted estimator $\tilde{Y} = \frac{1}{2}Y_1 + \frac{1}{3}Y_2 + \frac{1}{6}Y_3$.

a. Show that $\tilde{Y}$ is a linear estimator. (5%)

b. Show that $\tilde{Y}$ is an unbiased estimator. (5%)

c. Find the variance of $\tilde{Y}$ and compare it to the variance of the sample mean $\bar{Y}$. (5%)

d. Is $\tilde{Y}$ as good an estimator as $\bar{Y}$? (5%)