※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、選擇題 50 分(每題五分)

1. A theorem that allows us to use the normal probability distribution to approximate the sampling distribution of sample means and sample proportions whenever the sample size is large is known as the
   (a) Approximation Theorem
   (b) Normal Probability Theorem
   (c) Central Limit Theorem
   (d) Central Normality Theorem

2. Let A and B be events such that $P(A)=0.5$, $P(B)=0.5$, and $P(A^c \cap B^c)=1/3$, where $P(A^c)=1-P(A)$ and $P(B^c)=1-P(B)$. Then $P(A \cup B^c)=$
   (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{5}{6}$

3. Consider the following probability density function:

$$f(x)=\begin{cases} kx, & 0 \le x < 2, \\ k(4-x), & 2 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

What is the variance of X ?
(a) 1/3　(b) 2/3　(c) 4/3　(d) 5/3

4. Let $X_1, X_2, \cdots, X_5$ be a random sample of size 5 from $N(0, \sigma^2)$. Find the constant C so that $C(X_1-X_2)/\sqrt{X_3^2+X_4^2+X_5^2}$ has a $t-$distribution.
   (a) 0.5　(b) 0.6667　(c) 0.8165　(d) 1.2247

5. The probability density function of a random variable Y is $f(y)=k/y^3$, for $1<y<\infty$. What is the value of k?
   (a) 0.5　(b) 1.0　(c) 1.5　(d) 2

6. Let X and Y be two continuous random variables with joint density functio

$$f(x,y)=\begin{cases}4x & 0<x<\sqrt{y}<1\\ 0 & \text{otherwise}\end{cases}$$

What is the marginal density function of Y?

(a) $2y$　(b) $2y^2$　(c) y^2　(d) $2\sqrt{y}$

7. An analysis of personal loans at Taiwan Bank revealed the following facts: 10% of all personal loans are in default (D), 20% of those in default are homeowners $(H\mid D)$, and 70% of those not in default are homeowner $(H\mid \overline{D})$. If one of the personal loans is selected at random, $P(D\mid H)=$

(a) 0.02　(b) 0.63　(c) 0.03　(d) 0.18

8. Find ρ for X and Y if $f(x,y)=x+y$ for $0<x<1,\ 0<y<1,$ is the joint density of X and Y.

(a) 1　(b) $\frac{1}{11}$　(c) $-\frac{1}{11}$　(d) $\frac{7}{12}$

9. If a card is drawn from a desk of playing cards, what is the probability that it will be a jack or a ten ?

(a) $\frac{1}{13}$　(b) $\frac{2}{13}$　(c) $\frac{3}{13}$　(d) $\frac{4}{13}$

10. A research worker wishes to estimate the mean of a population using a sample large enough that the probability will be 0.95 that the sample mean will not differ from the population mean by more than 25 percent of the standard deviation. How large a sample should he take ? (for standard normal x, $p(-1.96<x<1.96)=0.95$)

(a) 59　(b) 60　(c) 61　(d) 62

二、非選擇題 50 分

1.(10%) Find a lower bound on $\Pr(-3<X<3)$ where $\mu=E(X)=0$ and $Var(X)=\sigma^2=1$. (Hint: Use Chebyshev's Inequality)

2. (10%) If x follows a discrete uniform distribution, i. e. $F(x)=\frac{1}{N}$ for $x=1,2,\cdots,N$,

find $E(x)$ and $Var(x)$.

3. (10%) Tom and Joe like to throw darts. Tom throws 100 times and hits the target 54 times; Joe throws 100 times and hits the target 49 times. Find a 95 percent confidence interval for p_1-p_2 where p_1 represents the true proportion of hit in Tom's tosses, and p_2 represents the true proportion of hits in Joe's tosses.

4. (10%) A random variable $X\sim Exp\left(\frac{1}{\lambda}\right)$, find the value of $E(X\mid X>1)$.

5. (10%) The probability density function of a random variable X is $f(x)=\begin{cases}\frac{1}{n}, & x=1,2,\cdots n\\ 0, & \text{otherwise}\end{cases}$,

Prove that

$$E\left[kX^{k-1}+\binom{k}{2}X^{k-2}+\cdots+1\right]=\frac{(n+1)^k-1}{n}.$$