

## 微積分

1. 已知  $y^2 - 6px = a, (p > 0)$ , 求  $\frac{dy}{dx}$  及通過  $(x_0, y_0)$  之切線方程式。(10%)
2.  $x = acost, y = bsint$ . 求 1)  $dy/dx$ ; 2) 當  $t = \pi/2$  時,  $dy/dx = ?$ 。(10%)
3. 設  $\int f(x)dx = F(x)$ . 試證 1)  $\int f(x-a)dx = F(x-a), a \in \mathbb{R}$ .  
2)  $\int f(x\gamma)dx = F(x\gamma)/\gamma, \gamma \neq 0$ 。(10%)
4. 設  $f(x, y) = x^2 + xy - y^2$ . 試求  $f(x, y)$  在  $(1, -2)$  的泰勒(Taylor)展開式。  
(10%)
5. Evaluate  $\int_0^3 x\sqrt{1+x}dx$ . (10%)
6. A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by
 
$$f(x) = \begin{cases} -x^2 & \text{when } x < 0 \text{ and} \\ x^2 & \text{when } x \geq 0. \end{cases}$$
 Determine whether 1)  $f$  is continuous at 0, 2)  $f'(0)$  exists. (10%)
7. Let  $\{x_n\} (n = 1, 2, \dots)$  be an increasing sequence, and assume that it is bounded from above. Show that the least upper bound  $b$  of the set  $\{x_n\} (n = 1, 2, \dots)$  is the limit of the sequence. (20%)
8. Show that  $\lim_{n \rightarrow \infty} \frac{2n}{n-1} = 2$ . (20%)