

一、選擇題 (一題 5 分, 共計 50 分)

1. If $f(x) = \begin{cases} c & \text{if } x = -3 \\ \frac{9-x^2}{4-(x^2+7)^{1/2}} & \text{if } |x| < 3 \\ d & \text{if } x = 3 \end{cases}$, then f is continuous on $[-3, 3]$ provided

- a) $c = 8$ and $d = -8$ b) $c = d = 8$ c) $c = -8, d = 8$ d) $c = d = 6$ e) $c \neq d$

2. If $f(x) = \begin{cases} |x| & \text{if } |x| \leq 1 \\ 2 - |x| & \text{if } x > 1 \end{cases}$, then which of the following is true?

- a) f is not continuous at $x = 1$ but it is differentiable at $x = 1$
 b) f is differentiable at all x
 c) f is continuous but not differentiable at $x = 0$
 d) f is continuous and differentiable at $x = 1$
 e) none of these

3. A rectangle has its vertices on the x -axis, the y -axis, the origin, and the graph of $y = 4 - x^2$. Find the maximum possible area for such a rectangle.

- a) $\frac{8\sqrt{3}}{9}$ b) $\frac{16\sqrt{3}}{9}$ c) $\frac{32\sqrt{3}}{9}$ d) $2\sqrt{3}$ e) none of these

4. $\int_2^{\pi} \sin x(1 + \sqrt{\cos x})^2 dx$ equals

- a) $\frac{17}{6}$ b) 3 c) $\frac{19}{6}$ d) $-\frac{19}{6}$ e) none of these

5. The area bounded by the curves $y + x^3 = 0$, $y = \sqrt{x}$, and $3y + 7x = 10$ equals

- a) $\frac{53}{3}$ b) $\frac{53}{4}$ c) $\frac{53}{6}$ d) $\frac{51}{5}$ e) none of these

6. The volume of the solid generated by revolving the region bounded by the curve $y = x^3$, $x = -2$, and $y = 0$ about the x -axis equals

- a) $\frac{128\pi}{5}$ b) 18π c) $\frac{120\pi}{2}$ d) $\frac{128\pi}{7}$ e) none of these

7. The series $\sum_{n=1}^{\infty} \frac{1}{\ln(n)}$

- a) converges by comparison to $a_n = \frac{1}{n}$ b) diverges by comparison to $a_n = \frac{1}{n}$
 c) converges by the integral test d) converges by comparison to $a_n = \frac{1}{n^2}$
 e) none of these

8. The interval of convergence of $s = \frac{x}{2^2 \cdot 3} + \frac{x^2}{3^2 \cdot 3^2} + \frac{x^3}{4^2 \cdot 3^3} + \dots$ is
 a) $(-3, 3)$ b) $[-3, 3)$ c) $(-3, 3]$ d) $[-3, 3]$ e) none of these
9. What dose $\iint_R (x^3 - y^3) dA$ equal where R is bounded by $y = x^3, x = 0$ and $y = 0$?
 a) $-\frac{45}{364}$ b) $\frac{45}{364}$ c) $\frac{9}{46}$ d) $\frac{1}{5}$ e) none of these
10. The volume of the solid in the first octant bounded by $x + y = 4$ and $z = xy$ is
 a) $\frac{44}{3}$ b) 15 c) $\frac{42}{3}$ d) $\frac{43}{3}$ e) none of these

二、計算或證明題 (一題 10 分, 共計 50 分), 需寫出詳細計算或證明過程, 否則扣分或不計分。

- Using the Definition of a Limit, Show that $\lim_{x \rightarrow c} \frac{1}{x} = \frac{1}{c}, c \neq 0$.
- Show that $f(x) = \sqrt{2x+1} + 2x$ is continuous at 3.
- Show that f satisfies the hypotheses of Rolle's theorem on the interval $[2, 6]$, and find all numbers c in $(2, 6)$ that satisfy the conclusion of Rolle's theorem for $f(x) = 2x^2 - 16x + 11$.
- A solid has as its base the region in the xy -plane bounded by the graphs of $y^2 = 4x$ and $x = 4$. If every cross section by a plane perpendicular to the y -axis is a semicircle, find the volume of the solid.
- Express $\iint_R (3x + 2y) dA$ over the triangular region with vertices $(-2, -2), (-1, -2), (-1, -1)$ as an iterated integral and find its value.