

(一) 選擇題一：(每題六分，共三十分。注意：請寫明演算過程，否則不予計分。)

- (1) You are performing a multiple regression of the forms: $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + e_i$. You have obtained the following data: Determine $\hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2$

Y	1	2	3	4
X ₁	-1	1	-1	1
X ₂	-1	-1	1	1

- (A) 3.5 (B) 4.0 (C) 4.5 (D) 5.0 (E) 5.5

- (2) A car company is monitoring the average number of defective cars produced by three assembly lines with equal levels of production. The number of defective cars per week has been recorded as follow: The total sum of squares about mean is 1156.11. Determine the *F*-value and the outcome of the test for equality of means at the 5% level.

								total
1	48	58	53	69	51	70		349
2	48	70	55	61	71	64	58	427
3	58	49	51	50	54	51		313

- (A) $F=2.16$; reject the null hypothesis. (B) $F=2.30$; do not reject the null hypothesis.
 (C) $F=2.45$; reject the null hypothesis. (D) $F=2.60$; do not reject the null hypothesis.
 (E) None of the above.

- (3) Two observations on *Y* were obtained for each of $X = 1, 2, 3, 4, 5$. A simple linear regression model fitted to these data resulted in the estimated regression line $\hat{Y} = 4 + X$ and following partial ANOVA table: Assuming that *Y* is normally distributed, a 95% prediction interval for *Y* when $X = 5$ is in the form $9 \pm K$. Determine *K*.

Source	SS	DF	MS
Regression		1	
Residual	10.00		
Total	30.00	9	

- (A) 1.4 (B) 2.0 (C) 2.6 (D) 2.9 (E) 3.3

- (4) Let \bar{X} be the sample mean from a normal distribution with variance 9. The $H_0: \mu = 100$ is rejected in favor of $H_1: \mu > 100$ if $\bar{X} > c$. If the significance level is 0.05, find the minimum sample size is needed to achieve a power of 0.5 when $\mu = 101$. The required sample size is the smallest integer larger than

- (A) $3(1.64)$ (B) $3(1.96)$ (C) $[3(1.64)]^2$ (D) $[3(1.96)]^2$ (E) $[3(1.64)]^{1/2}$

(背面仍有題目,請繼續作答)

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- (5) An advertiser uses simple linear regression to relate the daily number of ads(X) to the daily number of responses(Y). The data is as follow, and use least squares, you obtain the equation

$$\hat{Y} = -1.6 + 2.2X. \text{ Determine the value of the Durbin-Watson statistic.}$$

Day	Mon	Tue	Wed	Thu	Fri
X	1	2	3	4	5
Y	1	2	5	8	9

- (A) 1.4 (B) 2.0 (C) 2.6 (D) 2.9 (E) 3.2

(二) 選擇題二：(每題五分，共五十分。注意：可以不用寫明演算過程。)

- (1) Let Z_1, Z_2, Z_3 be independent normal random variable each with mean 0 and variance 1. Which of the following has a chi-square distribution with 1 degree of freedom.

(A) $\frac{Z_1^2 + Z_2^2}{2}$ (B) $\frac{(Z_1 + Z_2 - Z_3)^2}{3}$ (C) $Z_1^2 + Z_2^2 - Z_3^2$

(D) $(Z_1 + Z_2)^2 - Z_1^2$ (E) $(Z_1 + Z_2 - Z_3)^2$

- (2) A pair of dice is tossed 10 times in succession. What is the probability of observing no 7's and no 11's in any of the ten tosses?

(A) $\left(\frac{28}{36}\right)^{10}$ (B) $\left(\frac{30}{36}\right)^{10} \left(\frac{34}{36}\right)^{10}$ (C) $\left[1 - \left(\frac{6}{36}\right)\left(\frac{2}{36}\right)\right]^{10}$

(D) $1 - \left(\frac{8}{36}\right)^{10}$ (E) $\left[1 - \left(\frac{6}{36}\right)^{10}\right] \left[1 - \left(\frac{2}{36}\right)^{10}\right]$

- (3) Let X have the density function $f(x) = \frac{2x}{k^2}$ for $0 \leq x \leq k$. For what value of k is the variance of X equal to 2?

(A) 2 (B) 6 (C) 9 (D) 18 (E) 36

- (4) The following table represents the relative frequency of accidents per day in a city.

Accident	0	1	2	3	4 or more
Relative Frequency	.55	.20	.10	.10	.05

Which of the following statements are true?

- I. The mean and modal number of accidents are equal.
 II. The mean and median number of accidents are equal.
 III. The median and modal number of accidents are equal.

- (A) I only (B) II only (C) III only (D) I, II, and III (E) none of the above.

- (5) The hypothesis $H_0: \mu = 0$ v.s. $H_1: \mu = 1$ using a one-sample t -statistic when sampling from a normal distribution. Which of the following statements are true?
- $\beta = 1 - \alpha$.
 - If α is held constant and the sample size n is increasing, then β decreases.
 - If n is fixed and α is increasing, then β decreases.
- (A) I only (B) II only (C) I and III only (D) II and III only (E) none of the above.

- (6) Let X have moment generating function $M(t) = \frac{1}{(1-t)^2}$ for $t < 1$. Find $E(X^3)$.
- (A) -24 (B) 0 (C) 0.25 (D) 24 (E) 48.

- (7) Let X have the density function $f(x) = e^{x-2}$, for $x < 2$. What is the 75th percentile of X ?
- (A) $2 + \ln .75$ (B) $2 - \ln .75$ (C) $\ln(1 + .75e^2)$ (D) $-\ln(1 - .75e^2)$ (E) $2 + \ln .25$

- (8) A random sample with size 9 from a normal distribution. Which of following are the end points for a 90% confidence interval for μ ?

(A) $\bar{x} \pm \frac{0.233}{\sqrt{2}} \sqrt{\sum_{i=1}^9 (x_i - \bar{x})^2}$ (B) $\bar{x} \pm \frac{0.273}{\sqrt{2}} \sqrt{\sum_{i=1}^9 (x_i - \bar{x})^2}$ (C) $\bar{x} \pm \frac{0.305}{\sqrt{2}} \sqrt{\sum_{i=1}^9 (x_i - \bar{x})^2}$

(D) $\bar{x} \pm \frac{0.310}{\sqrt{2}} \sqrt{\sum_{i=1}^9 (x_i - \bar{x})^2}$ (E) $\bar{x} \pm \frac{0.385}{\sqrt{2}} \sqrt{\sum_{i=1}^9 (x_i - \bar{x})^2}$

- (9) For two paired samples with a sample of size n , the degree of freedom for t -statistic is
- (A) $2n - 1$ (B) $2n - 2$ (C) $n - 1$ (D) $n - 2$ (E) $n - 4$

- (10) A production process is working normally if the average weight of a manufactured steel bar is at least 1.3 pounds. A sample of 50 steel bars yields a mean of 1.26 pounds with a standard deviation of .10 pounds. The question is whether there is sufficient evidence to indicate that the production process needs adjusting, using 0.05 significance level. The p -value for this test is
- (A) .0126 (B) .0023 (C) .0011 (D) .0252 (E) .0046

(背面仍有題目,請繼續作答)

(三) For a randomized block model $Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$, ε_{ij} are independent, identical distributed

from normal distribution with mean 0 and variance σ^2 . $i = 1, 2, \dots, a$, $j = 1, 2, \dots, b$. And

$$SST = \sum_{i=1}^a \sum_{j=1}^b (Y_{ij} - \bar{Y})^2, \quad SSF = b \times \sum_{i=1}^a (\bar{Y}_i - \bar{Y})^2, \quad SSB = a \times \sum_{j=1}^b (\bar{Y}_j - \bar{Y})^2,$$

$$SSE = \sum_{i=1}^a \sum_{j=1}^b (Y_{ij} - \bar{Y}_i - \bar{Y}_j + \bar{Y})^2. \quad \text{Where } \bar{Y} = \sum_{i=1}^a \sum_{j=1}^b Y_{ij} / ab, \quad \bar{Y}_i = \sum_{j=1}^b Y_{ij} / b, \quad \bar{Y}_j = \sum_{i=1}^a Y_{ij} / a.$$

Show that $SST = SSF + SSB + SSE + SSE$. And under the null hypothesis that all α_i and β_j equal zero, what are the distributions of SST , SSF , SSB and SSE . (二十分)

(四) 附表：

$$t_{7,0.025} = 2.365, \quad t_{8,0.025} = 2.306, \quad t_{9,0.025} = 2.262, \quad t_{7,0.05} = 1.895, \quad t_{8,0.05} = 1.860, \quad t_{9,0.05} = 1.833,$$

$$t_{7,0.1} = 1.415, \quad t_{8,0.1} = 1.397, \quad t_{9,0.1} = 1.383, \quad F_{2,16,0.025} = 4.69, \quad F_{2,17,0.025} = 4.62, \quad F_{2,19,0.025} = 4.51,$$

$$F_{2,16,0.05} = 3.63, \quad F_{2,17,0.05} = 3.59, \quad F_{2,19,0.05} = 3.52.$$

標準常態分配表：

表中的數值代表介於平均數與 z 之間的面積

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4986	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990