

一. 選擇題: 60% (每題 6 分)

1. If two functions  $f$  and  $g$  are continuous at a real number  $c$ , then which function is also continuous at  $c$ : (a) the sum  $f + g$  (b) the difference  $f - g$  (c) the product  $f \cdot g$  (d) the quotient  $\frac{f}{g}$ , provide  $g(c) \neq 0$  (e) all of them

2. The hyperbolic sine function is denoted by  $\sinh$ . Find the derivative of the inverse function of it. That is, let  $y = \sinh^{-1} x$ , find  $\frac{dy}{dx} =$  (a)  $\frac{1}{\sqrt{x^2+1}}$  (b)  $\frac{x}{\sqrt{x^2+1}}$

(c)  $\frac{1}{\sqrt{x^2+x}}$  (d)  $\frac{x^2}{\sqrt{x^2+1}}$  (e)  $\frac{x}{\sqrt{x^2+x}}$

3. Evaluate  $\lim_{M \rightarrow \infty} \int_0^M \frac{dx}{x^4+4} = c$ ,  $c$  is a constant. What is the value of  $c$ ? (a)  $\frac{\pi}{2}$  (b)

$\frac{\pi}{4}$  (c)  $\frac{\pi}{6}$  (d)  $\frac{\pi}{8}$  (e)  $\frac{\pi}{12}$

4. What is the value of  $\int_0^1 \frac{1-e^{-x^2}}{x^2} dx$ ? (精確到第三位) (a) 0.710 (b) 0.637 (c) 0.231 (d) 0.425 (e) 0.862

5. Find the limits of  $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$  (a) 1 (b) 0 (c)  $\infty$  (d)  $-\infty$  (e) none

6.  $\varepsilon = 0.01$ , find a minimum positive integer  $N$ . For all  $n > N$ ,  $\left| \frac{3n+2}{n-1} - 3 \right| < \varepsilon$ ,

$N =$  (a) 302 (b) 402 (c) 502 (d) 602 (e) 702

7.  $u_{n+1} = \sqrt{u_n+1}$ ,  $u_1 = 1$ . The value of  $\lim_{n \rightarrow \infty} u_n =$  (a)  $1+\sqrt{5}$  (b)  $\frac{1}{2}(1+\sqrt{3})$

(c)  $\frac{1}{2}(1+\sqrt{5})$  (d)  $\frac{1}{2}(\sqrt{3}+\sqrt{5})$  (e)  $\frac{1}{3}(2+\sqrt{5})$

(背面仍有題目, 請繼續作答)

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8.  $a = \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$ ,  $b = \lim_{x \rightarrow 0^+} \left( \frac{1}{e^x - 1} - \frac{1}{x} \right)$ . What is the value of  $(a, b)$  (a)

$\left(\frac{1}{2}, \frac{1}{2}\right)$  (b)  $\left(-\frac{1}{2}, \frac{1}{2}\right)$  (c)  $\left(-\frac{1}{2}, -\frac{1}{2}\right)$  (d)  $\left(\frac{1}{2}, -\frac{1}{2}\right)$  (e)  $\left(1, \frac{1}{2}\right)$

9. Let  $y = x \arcsin(x)$ ,  $z = \ln|4 + 5x - 2x^3|$ . Find  $\frac{dy}{dx}$  and  $\frac{dz}{dx}$

(a)  $\arcsin(x) + \frac{1}{\sqrt{1-x^2}} + C_1, \frac{5-6x^2}{4+5x-2x^3} + C_2$

(b)  $\arcsin(x) + \frac{1}{\sqrt{1-x^2}} + C_1, \frac{|5-6x^2|}{4+5x-2x^3} + C_2$

(c)  $\arcsin(x) + \frac{x}{\sqrt{1-x^2}} + C_1, \frac{5-6x^2}{|4+5x-2x^3|} + C_2$

(d)  $\arcsin(x) + \frac{1}{\sqrt{1-x^2}} + C_1, \left| \frac{5-6x^2}{4+5x-2x^3} \right| + C_2$

(e)  $\arcsin(x) + \frac{x}{\sqrt{1-x^2}} + C_1, \frac{5-6x^2}{4+5x-2x^3} + C_2$  ( $C_1$  and  $C_2$  are constants)

10.  $V$  is the volume of the region beneath the surface  $z = xy^2 + y^3$  and over the rectangle  $R = \{(x, y) : 0 \leq x \leq 2 \text{ and } 1 \leq y \leq 3\}$ . What is the value of  $V$ ?

(a)  $\frac{152}{3}$  (b)  $\frac{161}{3}$  (c)  $\frac{172}{3}$  (d)  $\frac{292}{3}$  (e)  $\frac{224}{3}$

二. 計算證明題:40%

1. If  $y = x^x$  and  $x > 0$ , Find  $\frac{dy}{dx}$  (10%)

2. Evaluate the following functions:

a.  $\int \left( \frac{5x^3 - 3x^2 + 7x - 3}{(x^2 + 1)^2} + x \cdot 3^{x^2} \right) dx$  (5%)

b.  $\int (x^2 e^{2x} + e^x \cos x) dx$  (5%)

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3. Determine whether the series is convergent or divergent ?

a. 
$$\sum_{n=1}^{\infty} \frac{n^n}{n!} \quad (5\%)$$

b. 
$$\sum_{n=1}^{\infty} \frac{2^{3n+1}}{n^n} \quad (5\%)$$

4. Pick  $a_0$  and  $a_1$ . For  $n \geq 2$ , compute  $a_n$  recursively, so that

$$n(n-1)a_n = (n-1)(n-2)a_{n-1} - (n-3)a_{n-2}$$

Evaluate  $\sum_{n=0}^{\infty} a_n$ . (10%)