

※本考題附有常態分配及 t 分配表

選擇題: (50%)

1. (5%) If X is a random variable with mean 33 and variance 16, what is the lower bound for $P(23 < X < 43)$ and upper bound for $P(|X - 33| \geq 14)$. (Hint: Use Chebyshev's inequality to find the answer) (a) 0.54, 0.082 (b) 0.84, 0.082 (c) 0.62, 0.018 (d) 0.16, 0.018 (e) 0.54, 0.38
2. (5%) If 10 fair dice are rolled, find the approximate probability that the sum obtained is between 30 and 40. (Hint: Use central limit theorem to find the answer) (a) 0.21 (b) 0.32 (c) 0.46 (d) 0.65 (e) 0.84
3. (5%) Assume that IQ scores for a certain population are approximately $N(\mu, 100)$. To test $H_0: \mu = 110$ against the one-sided alternative hypothesis $H_1: \mu > 110$, we take a random sample of size $n = 16$ from this population and observe $\bar{x} = 113.5$. Do we accept or reject H_0 at 5% significance level? What is the p -value of this test? (a) Reject H_0 , p -value=0.0808 (b) Do not reject H_0 , p -value=0.0808 (c) Reject H_0 , p -value=0.0533 (d) Do not reject H_0 , p -value=0.0533 (e) Reject H_0 , p -value=0.0418.
4. (5%) A 5-card poker hand is said to be a full house if it consists of 3 cards of the same denomination and 2 cards of the same denomination. (that is, a full house is three of a kind plus a pair.) What is the probability of being dealt a full house? (a) 0.025 (b) 0.0033 (c) 0.0014 (d) 0.0088 (e) 0.0020
5. (5%) What is the distribution that has the same mean and variance? (a) Normal Distribution (b) T Distribution (c) Poisson Distribution (d) Geometric Distribution (e) none
6. (5%) Let $Y_1 < Y_2 < \dots < Y_{19}$ be the order statistics of a random sample of size $n = 19$ from the exponential distribution with mean θ . What is the p.d.f of Y_1 ? What is the value of $E[F(Y_1)]$, where F is the distribution function of the exponential distribution. (a) $g_1(y) = 18(e^{-y/\theta})^{19} \frac{1}{\theta} e^{-y/\theta}$ where $0 < y < \infty, 1/19$ (b) $g_1(y) = 19(e^{-y/\theta})^{19} \frac{1}{\theta} e^{-2y/\theta}$ where $0 < y < \infty, 1/19$ (c) $g_1(y) = 19(e^{-y/\theta})^{18} \frac{1}{\theta} e^{-y/\theta}$

(背面仍有題目, 請繼續作答)

where $0 < y < \infty$, $1/20$ (d) $g_1(y) = 19(e^{-2y/\theta})^{18} \frac{1}{\theta} e^{-2y/\theta}$ where $0 < y < \infty$, $1/20$

7. (5%) Suppose we have 3 cards identical in form except that both sides of the first card are colored red, both sides of the second card are colored blacked, and one side of the third card is colored red and the other side black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black? (a) 1/3 (b) 1/2 (c) 2/3 (d) 1/4 (e) 1/6
8. (5%) Let X and Y equal the lengths in inches of a foot and a hand, respectively. The following measurements were made on 15 woman.

x	y	x	y	x	y
9.00	6.50	10.00	7.00	9.25	7.00
8.50	6.25	9.50	6.50	10.00	7.50
9.25	7.25	9.00	7.00	10.00	7.25
9.75	7.00	9.25	7.00	9.75	7.25
9.00	6.75	9.50	7.00	9.50	7.25

What is the least squares regression line for these data?

- (a) $\hat{y} = 1.896 + 0.538x$ (b) $\hat{y} = 0.538 + 1.896x$ (c) $\hat{y} = 0.896 + 1.538x$ (d) $\hat{y} = 0.538 - 1.896x$ (e) $\hat{y} = 1.896 - 0.538x$

9. (5%) Thirteen tons of cheese is stored in some old gypsum mines, including "22-pound" wheels (label weight). A random sample of $n=9$ of these wheels yielded the following weights in pounds:

21.5 18.95 18.55 19.40 19.15
22.35 22.90 22.20 23.10

Assuming that the distribution of the weights of the wheels of cheese is $N(\mu, \sigma^2)$, which is the following answer is a 95% confidence interval for μ ?

- (a) [25.13, 28.43] (b) [12.34, 15.33] (c) [33.23, 35.72] (d) [24.23, 27.65] (e) [19.47, 22.33]

10. (5%) Let $f(x) = xe^x$, $0 < x < 1$ be the p.d.f of X . What is the value of μ (mean) and σ^2 (variance) (a) $e-1$, $e+2-e^2$ (b) $1-e$, e^2-e+2 (c) $e-2$, $2+2e-e^2$ (d) $2-e$, e^2+2e+2 (e) $e-2$, e^2+2e+2

計算證明題: (50%)

1. (10%) The correlation of two random variables X and Y , denoted by $\rho(X, Y)$, is defined, as long as $\text{Var}(X)$ and $\text{Var}(Y)$ is positive, by

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$$

Show that $-1 \leq \rho(X, Y) \leq 1$.

2. (10%) Show that $\text{Var}(x) = E[\text{Var}(X | Y)] + \text{Var}(E[X | Y])$.

3. (10%) X is a random variable. Show that if $f(x)$ is a convex function, then

$$E[f(X)] \geq f(E[X])$$

provided that the expectations exist and are finite.

4. (10%) If the moment-generating function of X is

$$M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t},$$

find the mean, variance, p.d.f. of X , and the probability-generating function.

5. (10%) Let X_1, X_2, \dots, X_n be a random sample from a normal distribution.

(a)(6%) show that an unbiased estimator for σ is cS , where

$$c = \frac{\sqrt{n-1}\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{2}\Gamma\left(\frac{n}{2}\right)}. \quad (\Gamma(t) = \int_0^{\infty} e^{-y} y^{t-1} dy.)$$

- (b)(4%) find the value of c when $n=5, n=6$.

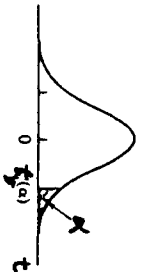
(背面仍有題目, 請繼續作答)

TABLE 1 STANDARD NORMAL PROBABILITIES



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
10	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
11	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
12	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
13	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
14	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
15	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
16	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
17	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
18	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
19	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
20	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
21	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
22	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
23	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
24	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
25	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
26	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
27	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
28	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
29	.9981	.9982	.9982	.9983	.9984	.9985	.9985	.9986	.9986	.9986
30	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
31	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
32	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
33	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
34	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
35	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998

TABLE 2 STUDENT'S t-DISTRIBUTION CRITICAL POINTS



d.f.	.250	.100	.050	.025	.010	.005	.0025	.001
1	1.000	1.078	1.314	1.706	3.182	3.182	3.182	3.182
2	.816	1.088	1.286	1.960	2.924	2.924	2.924	2.924
3	.765	1.038	1.250	1.833	2.747	2.747	2.747	2.747
4	.741	1.033	1.232	1.761	2.577	2.577	2.577	2.577
5	.727	1.029	1.225	1.746	2.571	2.571	2.571	2.571
6	.718	1.026	1.220	1.733	2.548	2.548	2.548	2.548
7	.711	1.024	1.218	1.724	2.537	2.537	2.537	2.537
8	.706	1.022	1.216	1.717	2.528	2.528	2.528	2.528
9	.703	1.021	1.215	1.712	2.521	2.521	2.521	2.521
10	.701	1.020	1.214	1.708	2.516	2.516	2.516	2.516
11	.699	1.019	1.213	1.705	2.512	2.512	2.512	2.512
12	.697	1.018	1.212	1.702	2.509	2.509	2.509	2.509
13	.695	1.017	1.211	1.699	2.506	2.506	2.506	2.506
14	.694	1.016	1.210	1.697	2.504	2.504	2.504	2.504
15	.692	1.015	1.209	1.695	2.502	2.502	2.502	2.502
16	.691	1.014	1.208	1.693	2.500	2.500	2.500	2.500
17	.689	1.013	1.207	1.691	2.498	2.498	2.498	2.498
18	.688	1.012	1.206	1.689	2.496	2.496	2.496	2.496
19	.687	1.011	1.205	1.687	2.494	2.494	2.494	2.494
20	.686	1.010	1.204	1.686	2.492	2.492	2.492	2.492
21	.685	1.009	1.203	1.684	2.490	2.490	2.490	2.490
22	.684	1.008	1.202	1.683	2.488	2.488	2.488	2.488
23	.683	1.007	1.201	1.682	2.486	2.486	2.486	2.486
24	.682	1.006	1.200	1.681	2.484	2.484	2.484	2.484
25	.681	1.005	1.199	1.680	2.482	2.482	2.482	2.482
26	.680	1.004	1.198	1.679	2.480	2.480	2.480	2.480
27	.679	1.003	1.197	1.678	2.478	2.478	2.478	2.478
28	.678	1.002	1.196	1.677	2.476	2.476	2.476	2.476
29	.677	1.001	1.195	1.676	2.474	2.474	2.474	2.474
30	.676	1.000	1.194	1.675	2.472	2.472	2.472	2.472
40	.674	1.000	1.193	1.674	2.470	2.470	2.470	2.470
60	.673	1.000	1.192	1.673	2.468	2.468	2.468	2.468
120	.672	1.000	1.191	1.672	2.466	2.466	2.466	2.466
∞	.671	1.000	1.190	1.671	2.464	2.464	2.464	2.464