

編號: G 360 系所: 財務金融研究所

科目: 微積分

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

一、選擇題: 50% (每題 5 分)

1. Evaluate $\int \frac{x^5}{x^4 - 2x^2 + 1} dx =$

(a) $\frac{1}{2}x^2 + \frac{1}{4}\log|x^4 - 2x^2 + 1| + \frac{1}{2}\log|x^2 - 1| + \frac{1}{2(x^2 - 1)}$.

(b) $\frac{1}{2}x^2 + \frac{1}{4}\log|x^4 - 2x^2 + 1| + \frac{1}{2}\log|x^2 - 1| - \frac{1}{2(x^2 - 1)}$.

(c) $\frac{1}{2}x^2 + \frac{1}{4}\log|x^4 - 2x^2 + 1| - \frac{1}{2}\log|x^2 - 1| + \frac{1}{2(x^2 - 1)}$.

(d) $\frac{1}{2}x^2 - \frac{1}{4}\log|x^4 - 2x^2 + 1| - \frac{1}{2}\log|x^2 - 1| - \frac{1}{2(x^2 - 1)}$.

(e) $\frac{1}{2}x^2 - \frac{1}{4}\log|x^4 - 2x^2 + 1| + \frac{1}{2}\log|x^2 - 1| + \frac{1}{2(x^2 - 1)}$.

2. Evaluate $\lim_{t \rightarrow 0} \frac{e^{2t} - e^t - t}{t^2} =$ (a) 0 (b) ∞ (c) 1 (d) $\frac{3}{2}$ (e) $\frac{1}{2}$

3. Find the value $\sum_{k=1}^{\infty} \frac{2^k}{(2^{k+1} - 1)(2^k - 1)} =$ (a) 1 (b) 0 (c) -1 (d) ∞ (e) $-\infty$

4. Find the value $\sqrt{5 + \sqrt{5 + \sqrt{5 + \dots}}} =$ (a) $\frac{1 + \sqrt{22}}{2}$ (b) $\frac{2 - \sqrt{23}}{2}$ (c) $\frac{4 + \sqrt{19}}{3}$

(d) $\frac{-1 + \sqrt{43}}{2}$ (e) $\frac{1 + \sqrt{21}}{2}$

5. Using differentials, find the approximate error in the volume of a sphere of radius r in. caused by a deviation in the radius by 1 per cent. What is the relative (or percentage) error? (a) 1% (b) 2% (c) 3% (d) 4% (e) 5%6. The complete solution of $(D^2 + D + 1)y = 2\sin 3x$ (D is the differential operator)is $\alpha \sin 3x + \beta \cos 3x + e^{-\frac{1}{2}x} \left(\cos \frac{1}{2}\sqrt{3}x + \sin \frac{1}{2}\sqrt{3}x \right)$, find $(\alpha, \beta) =$ (a)

$\left(\frac{16}{73}, -\frac{6}{73} \right)$ (b) $\left(\frac{16}{73}, \frac{6}{73} \right)$ (c) $\left(\frac{6}{73}, \frac{16}{73} \right)$ (d) $\left(-\frac{16}{73}, -\frac{6}{73} \right)$ (e) $\left(-\frac{6}{73}, \frac{16}{73} \right)$

7. Find the value $\lim_{x \rightarrow 0^+} x^x =$ (a) 0 (b) 1 (c) $+\infty$ (d) $-\infty$ (e) NaN

(背面仍有題目, 請繼續作答)

編號: G 360 系所: 財務金融研究所

科目: 微積分

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)8. Evaluate $\iint_D (a^2 - x^2 - y^2)^{\frac{1}{2}} dA$, where D is the disk of radius $\frac{a}{2}$ centered at

$(\frac{a}{2}, 0)$ in the XY -plane. (a) $\frac{2a^3}{3} \left(\frac{\pi}{2} - \frac{2}{3} \right)$ (b) $\frac{2a^3}{3} \left(\frac{\pi}{2} - \frac{1}{3} \right)$ (c) $\frac{2a^3}{3} \left(\frac{\pi}{2} + \frac{2}{3} \right)$ (d)

$\frac{2a^3}{3} \left(\frac{1}{2} - \frac{2}{3} \pi \right)$ (e) $\frac{a^3}{3} \left(\frac{\pi}{2} - \frac{2}{3} \right)$

9. Find the distance from the $(-1, 4, 2)$ to the plane $\phi: 2x - 3y + z - 7 = 0$

(a) $\frac{13}{14} \sqrt{14}$ (b) $\frac{19}{14} \sqrt{13}$ (c) $\frac{14}{19} \sqrt{14}$ (d) $\frac{19}{14} \sqrt{13}$ (e) $\frac{19}{14} \sqrt{14}$

10. $a_{n+1} = \frac{1}{2} \left(a_n + \frac{2}{a_n} \right)$, $a_1 = 2$, $\lim_{n \rightarrow \infty} a_n =$ (a) 2 (b) 1 (c) $\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$ (e) ∞

二、計算證明題: 50%

1. Evaluate the following functions:

a. Let $S = \left\{ (x, y, z) : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \right\}$. Find $\iiint_S (xy + xz + yz) dz dy dx$ (10%)b. Is $\sum_{n=2}^{\infty} \frac{1}{[\ln(\ln n)]^{n^n}}$ convergent? (5%)c. $f(x) = \int_0^x \ln(1+t) dt$, find $f(x)$. (5%)d. Let $y = x^{\sin x}$, find $\frac{d^2 y}{dx^2}$ (5%)

2. Find the limits of the following functions:

a. Show that $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right)$ (5%)b. Approximate $\sqrt{4.01}$ using differentials. (10%)3. Determine whether the series a_n is convergent or divergent? If a_n is convergent,find $\lim_{n \rightarrow \infty} a_n$.a. $a_n = (2n)^{\frac{1}{2n}}$ (5%)b. $a_n = \frac{n \cos(n\pi)}{2n-1}$ (5%)