本試題是否可以使用計算機: ☑可使用 , □不可使用 (請命題老師勾選)

一、選擇題 50分(每題五分)

1. Evaluate $\iint\limits_R f(x,y)\,dA$, where $f(x,y) = x + 2y$ and R is the rectangle defined by $1 \le x \le 4$ and $1 \le y \le 2$. (a) $\dfrac{15}{2}$ (b) 6 (c) $8\dfrac{1}{2}$ (d) $16\dfrac{1}{2}$

2. Suppose that x units of labor and y units of capital are required to produce $f(x,y) = 100x^{3/4}y^{1/4}$ units of a certain product (recall that this is a Cobb-Douglas production function). If each unit of labor costs \$200 and each unit of capital costs \$300, and a total of \$60,000 is available for production, determine how many units of capital should be used in used in order to maximize production.

(a) $x = 150, y = 100$ (b) $x = 200, y = \dfrac{200}{3}$ (c) $x = 225, y = 50$ (d) $x = 250, y = \dfrac{100}{3}$

3. Evaluate $\displaystyle\int \dfrac{(\ln x)^2}{2x}\,dx$ (a) $\dfrac{1}{6}x(\ln x)^2 + c$ (b) $\dfrac{1}{6}x(\ln x)^3 + c$ (c) $\dfrac{1}{6}x(\ln x^2) + c$ (d) $\dfrac{1}{6}(\ln x)^3 + c$

4. Find $\displaystyle\lim_{x \to 0}(1+x)^{\frac{1}{x}} = $ (a) 0 (b) 1 (c) ∞ (d) e

5. Find $\displaystyle\lim_{x \to \infty}(x^2 + a^2)^{\left(\frac{1}{x}\right)^2} = $ (a) 0 (b) 1 (c) ∞ (d) e

6. Find an equation of the tangent line to the graph of $y = \sqrt{4 - x^2}$ at the point $(1, \sqrt{3})$. (a) $y = -\sqrt{3}x + 2\sqrt{3}$ (b) $y = -2\sqrt{3}x + 3\sqrt{3}$ (c) $y = -\dfrac{\sqrt{3}}{3}x + \dfrac{4}{3}\sqrt{3}$ (d) $y = \dfrac{\sqrt{3}}{3}x + \dfrac{2}{3}\sqrt{3}$

7. Evaluate $\displaystyle\int_{-\infty}^{\infty} xe^{-x}\,dx = $ (a) 0 (b) 1 (c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$

8. Find the area between $y = \cos x$ and $y = \sin x$, $x \in [0, 2\pi]$ (a) 2 (b) $2\sqrt{2}$ (c) 4 (d) $4\sqrt{2}$

9. Find $\displaystyle\int \dfrac{\sec x}{2\tan x + \sec x - 1}\,dx = $ (a) $\log\left|\dfrac{\tan\frac{1}{2}x}{\tan\frac{1}{2}x+1}\right| + C$ (b) $\dfrac{1}{2}\log\left|\dfrac{\tan x}{\tan x + 2}\right| + C$ (c)

(背面仍有題目,請繼續作答)

$$\frac{1}{2}\log\left|\frac{\tan\frac{1}{2}x}{\tan\frac{1}{2}x+2}\right|+C \quad \text{(d)} \quad \frac{1}{2}\log\left|\frac{\tan x}{\tan\frac{1}{2}x+2}\right|+C$$

10. Evaluate $\displaystyle\int_0^1\int_{-2x}^{1-x}\int_0^{\sqrt{2-x^2}} z\,dz\,dy\,dx$ (a) 0 (b) -1 (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{3}$

二、非選擇題 50 分

1. (20%) Evaluate the following functions:

   a. $\displaystyle\int x^2 e^{\frac{1}{2}x}\,dx$ (5%)

   b. $\displaystyle\int\left(\frac{x^5+2}{x^2-1}\right)dx$ (5%)

   c. $\displaystyle\int x^3 e^x\,dx$ (5%)

   d. $\displaystyle\int\tan^6 x\,dx$ (5%)

2. (10%) Prove that for each positive integer n,

$$\left(1+\frac{1}{n}\right)^n \le e \le \left(1+\frac{1}{n}\right)^{n+1}$$

3. (10%) Estimate

$$\int_0^2\sqrt{4+x^3}\,dx$$

   (a) (5%) by the trapezoidal rule. Take $n=4$.

   (b) (5%) by the Simpson's rule. Take $n=2$.

4. (10%) Show that

$$\int_{-\infty}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}\,dx=1$$

   for any μ and for $\sigma>0$.