※ 考生請注意：本試題 □可　☑不可 使用計算機

1. (20%)　In terms of the entries a and b, when is A invertible?

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

2. (20%) An accident occurs at a point X that is uniformly distributed on a road of length L. At the same time of the accident, an ambulance is at a location Y that is also uniformly distributed on the road. Assuming that X and Y are independent, find the expected distance between the ambulance and the point of the accident.

3. (20%) Fibonacci numbers are defined by the recurrence relation $F_{k+2} = F_{k+1} + F_k$ with seed values $F_0 = 0$ and $F_1 = 1$. For instance, by adding $F_6 = 8$ and $F_7 = 13$ we reach $F_8 = 21$. Prove that the kth Fibonacci number is the nearest integer to $\frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^k$. (*hint*: eigenvalues may tell how fast the Fibonacci number grow)

4. (20%) Find the range of c and d values that make A and B positive definite.

$$A = \begin{bmatrix} c & 1 & 1 \\ 1 & c & 1 \\ 1 & 1 & c \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & d & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

5. (20%) The probability density function of an exponentially distributed random variable T is

f(t) $= \frac{1}{\theta} e^{-\frac{t}{\theta}}$, $0 \le t < \infty$. Assume that the life of a certain type of telecommunication component has an exponential distribution with a mean life of 500 hours. Suppose that a component has been in operation for 300 hours, calculate the probability that it will last for another 600 hours.