(1) Given a 3 by 3 matrix $A=\begin{bmatrix} 2 & 3 & 1 \\ 4 & -5 & 3 \\ -2 & 8 & -1 \end{bmatrix}$, find the lower triangular matrix L and upper triangular matrix U such that $A=LU$. (20%)

(2) A continuous complex-valued function is defined on the interval $[0,2\pi]$ with the inner product: $\langle f,g\rangle=\frac{1}{2\pi}\int_0^{2\pi} f(t)\overline{g(t)}\,dt$, where the bar denotes complex conjugation. Given i the imaginary number such that $i^2=-1$. Let $f_n(t)=e^{int}$, where $0\le t\le 2\pi$. Find $\langle f_m,f_n\rangle$, for any integer m,n. (20%)

(3) Given $A^2=\begin{bmatrix} -2 & -3 & 3 \\ -5 & 4 & 5 \\ -11 & -3 & 12 \end{bmatrix}$, find A. (20%)

(4) The time, in hours, it takes to locate and repair an electrical breakdown in a cell phone factory is a random number, X, whose density function is given by

$$f_X(x)=\begin{cases} 1 & \text{if } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

If the cost involved in a breakdown of duration x is x^3, what is the expected cost of such a breakdown? (20%)

(5) Suppose that 15 percent of the residents in Taiwan in a certain community have no cell phone, 20 percent have 1, 35 percent have 2, and 30 percent have 3 cell phones; and suppose, further, that for each resident, each cell phone is equally likely (and independently) to be a smartphone (S) or a traditional feature phone (T). (a) Please calculate the probability that a resident chosen at random from this community will have at least 1 traditional feature phone. (b) If we know that the resident chosen has exact 1 traditional feature phone, please compute the probability that this resident will also own at least 1 smartphone. (20%)