1. (10%) Let $X \sim Bin(m, p_1)$ and $Y \sim Bin(n, p_2)$ with X and Y independent variables. Assume that $\hat{p}_1 = X/m$ and $\hat{p}_2 = Y/n$. Show that $\hat{p}_1 - \hat{p}_2$ is an unbiased estimator of $p_1 - p_2$. Find the variance of $\hat{p}_1 - \hat{p}_2$.

2. Let $X_1, X_2, \ldots$ be independent and identically distributed (*iid*) random variables with $EX_i = \mu$ and $Var\, X_i = \sigma^2 < \infty$. Define $\bar{X}_n = (\frac{1}{n}) \sum_{i=1}^{n} X_i$. Then for every $\epsilon > 0$, state:
   (1) Weak Law of Large Numbers. (10%)
   (2) Strong Law of Large Numbers. (10%)
   (3) Assume that $X_1, X_2, \ldots$ are *iid* random variables whose moment generating functions exist in a neighborhood of 0. Let $G_n(X)$ denote the cumulative distribution function of $\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma}$. For any $-\infty < x < \infty$, state the Center Limit Theorem. (10%)

3. Answer the following two questions regarding mean value μ from a normal distribution:
   (1) (10%) Given a random sample $X_1, X_2, \ldots, X_n$ from a normal distribution $N(\mu, \sigma^2)$, derive the random interval that has the probability of $1 - \alpha$ to include the known mean μ.
   (2) (10%) Let X equal to the length of life of a product manufactured by a company. Assume that the distribution of X is $N(\mu, 1936)$. If a random sample of $n = 121$ products were tested until they failed, yielding a sample mean of $\bar{x} = 1789$ hours. Find the random interval that μ has the probability of 95% to include the known mean μ.

4. (20%) Let X_1 and X_2 have the joint probability density function $f(x_1, x_2) = \frac{x_1 + 2x_2}{18}$, $x_1 = 1,2$, $x_2 = 1,2$. Find the covariance of X_1 and X_2.

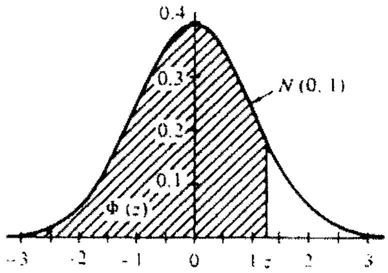
5. (20%) Let H_0 be the null hypothesis and β be the probability of type II error (involves not rejecting H_0 when H_0 is false). Denote $\beta(\mu') = P(H_0$ is not rejected when $\mu = \mu')$. Show that for any $\Delta > 0$, when the population distribution is normal and σ is known, the two-tailed test satisfies $\beta(\mu_0 - \Delta) = \beta(\mu_0 + \Delta)$, so that $\beta(\mu')$ is symmetric about μ_0.

編號： 312 **國立成功大學一〇一學年度碩士班招生考試試題**

系所組別： 電信管理研究所乙組

考試科目： 統計學 考試日期：0225，節次：2

The Normal Distribution



$$P(Z \le z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$[\Phi(-z) = 1 - \Phi(z)]$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

α	0.400	0.300	0.200	0.100	0.050	0.025	0.010	0.005	0.001
z_α	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090
$z_{\alpha/2}$	0.842	1.036	1.282	1.645	1.960	2.240	2.576	2.807	3.291