系所組別：電信管理研究所乙組

考試科目：線性代數

考試日期：0224，節次：2

※ 考生請注意：本試題不可使用計算機

(1) Let D^k denote the kth derivative operator. Find the general solution of the differential equation: $L(y)=0$, where $L=D^3-D^2-8D+12$. (20%)

(2) $A=\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$, find e^{tA}. (20%)

(3) Given an equation: $2x_1^2+4x_1x_2+5x_2^2+4x_1+13x_2-\frac{1}{4}=0$, transfer the equation into the form: $ap^2+bq^2=c$, where p, q are variables; and a, b, c are constants. Find the value of a, b, c and express p, q in terms of x_1 and x_2. (20%)

(4) Given a square matrix A, if $A^2=A$, prove that $(A+I)^k=I+(2^k-1)A$, where I is the identity matrix. (20%)

(5) $f(x)=\begin{vmatrix}1 & x & x^2 & x^3\\ 0 & 1 & 2x & 3x\\ 0 & 0 & 2 & 3\\ 1 & e^x & e^{2x} & e^{3x}\end{vmatrix}$, solve $f'(x)=0$. (20%)