※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. **(20%)** Consider the following simple linear regression model: $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, $i = 1 \dots n$ where

    Y_i is the value of the response variable in the i th trial

    β_0 and β_1 are parameters

    X_i is a known constant, the level of the predictor variable in the i th trial

    ε_i is a random error term with mean $E\{\varepsilon_i\} = 0$ and variance $\sigma^2\{\varepsilon_i\} = \sigma^2$; ε_i and ε_j are uncorrelated.

    Please use the method of least squares to derive the point estimators of β_0 and β_1, b_0 and b_1, respectively, that minimize $Q = \sum_{i=1}^{n}(Y_i - \beta_0 - \beta_1 X_i)^2$ for the given sample observations $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$.

2. **(20%)** The goal in Hypothesis Testing is to analyze a sample in an attempt to distinguish between population characteristics that are likely to occur and population characteristics that are unlikely to occur. Please clearly explain what are the two types of errors result in the process of testing hypothesis and define what is the power of a statistical test.

3. **(20%)** The results of a recent study regarding passenger gender and three transportation modes are shown in the following table.

Mode	Female	Male	Totals
Motorcycle	20	60	80
Bus	70	80	150
Car	30	40	70
Totals	**120**	**180**	**300**

    We are interested in determining whether or not transportation mode is independent of gender. Please compute the test statistic.

4. **(20%)** John passes through four traffic lights on his way to school, and each light is equally likely to be green or red, independently of the others. Suppose that each red light delays John by exactly two minutes. What is the variance of John's commuting time?

5. **(20%)** We assume that the number of bus arriving at the NCKU bus stop in any interval of length t is a Poisson random variable with parameter λt. Suppose that bus arriving at the NCKU bus stop at a rate of 2 per hour.

    (a) Find the probability that at least 3 buses arriving during the next 2 hours.

    (b) Find the probability distribution of the time, starting from now, until the next bus arrive.